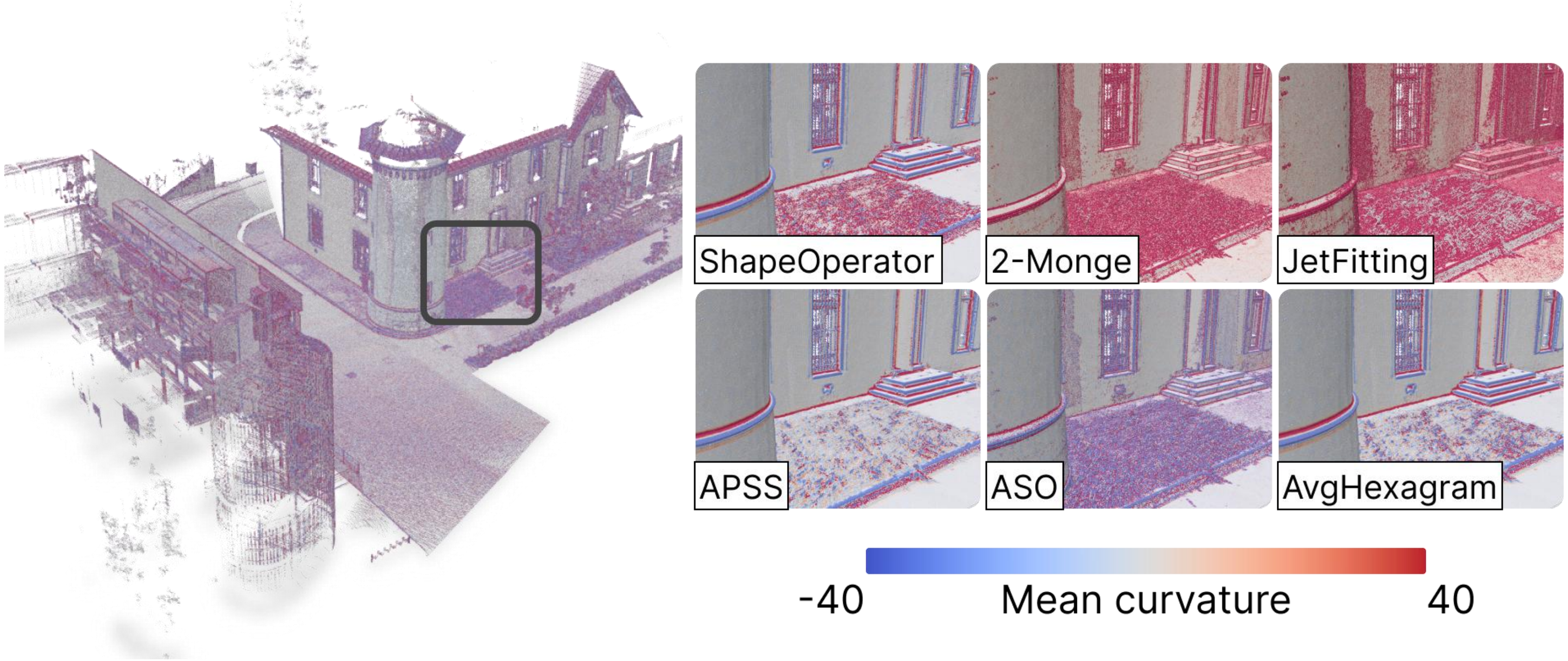
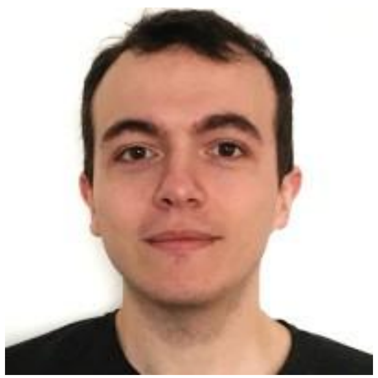
Survey on differential estimators for 3d point clouds



Survey on differential estimators for 3d point clouds

Léo Arnal--Anger[1],**Thibault Lejemble**[2], David Coeurjolly[3], Loïc Barthe[1], Nicolas Mellado[1]


[1] IRIT, Université de Toulouse, CNRS, Toulouse INP, UT, Toulouse, France

[2] Cerema, Toulouse, France

[3] CNRS, Université Claude Bernard Lyon 1, INSA Lyon, LIRIS, France

Context

01

Context

Survey on differential estimators for 3d point clouds

Context

Survey on differential estimators for 3d point clouds

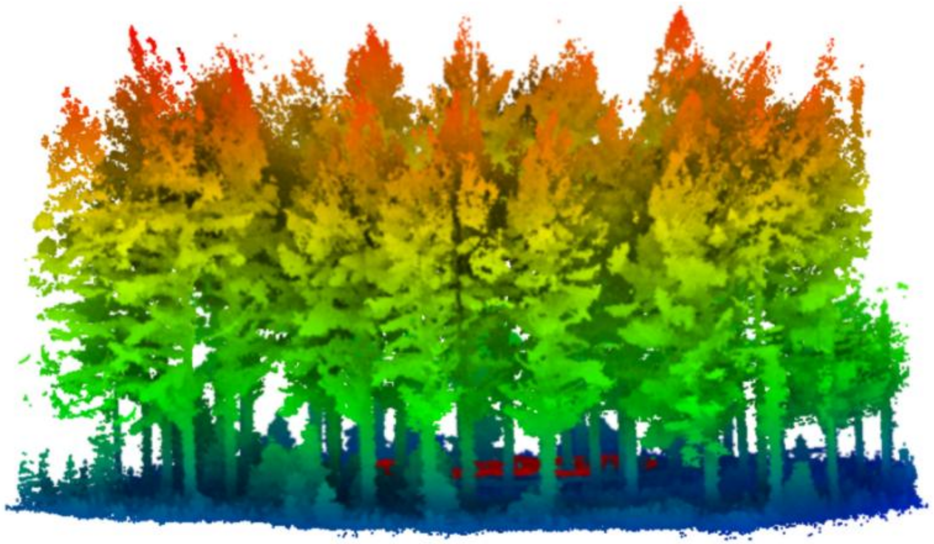
Context

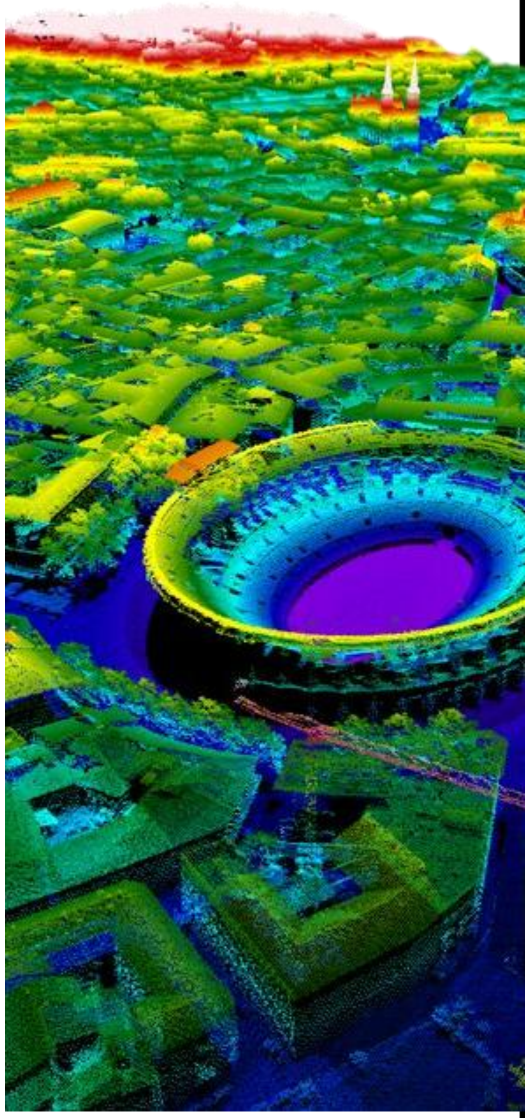
Survey on differential estimators for 3d point clouds

3D point clouds

- Capture real world geometry
- From real objects to virtual scenes







3D point clouds – Applications

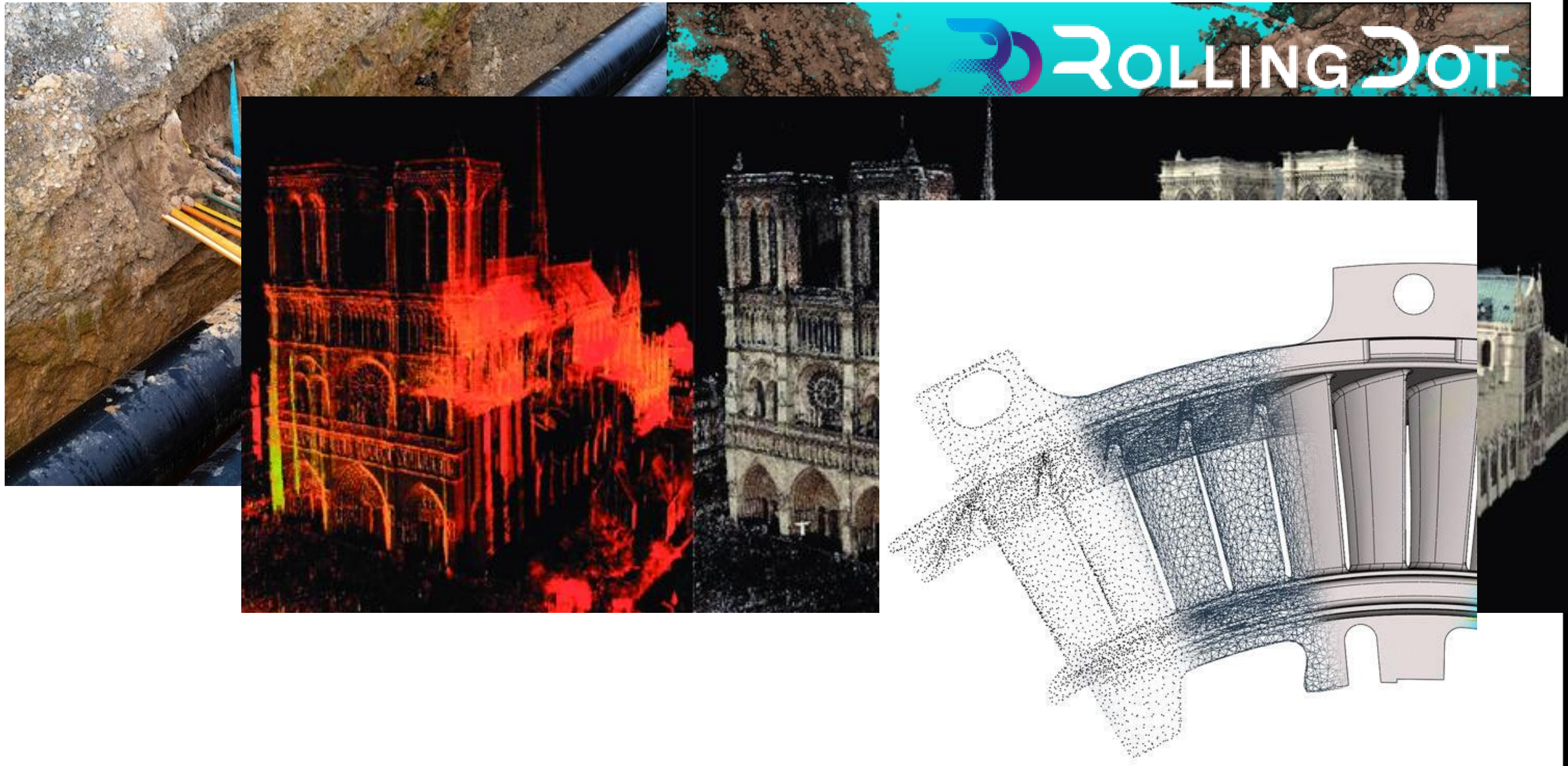
- Maintenance

3D point clouds – Applications

- Maintenance
- Cultural Heritage

3D point clouds – Applications

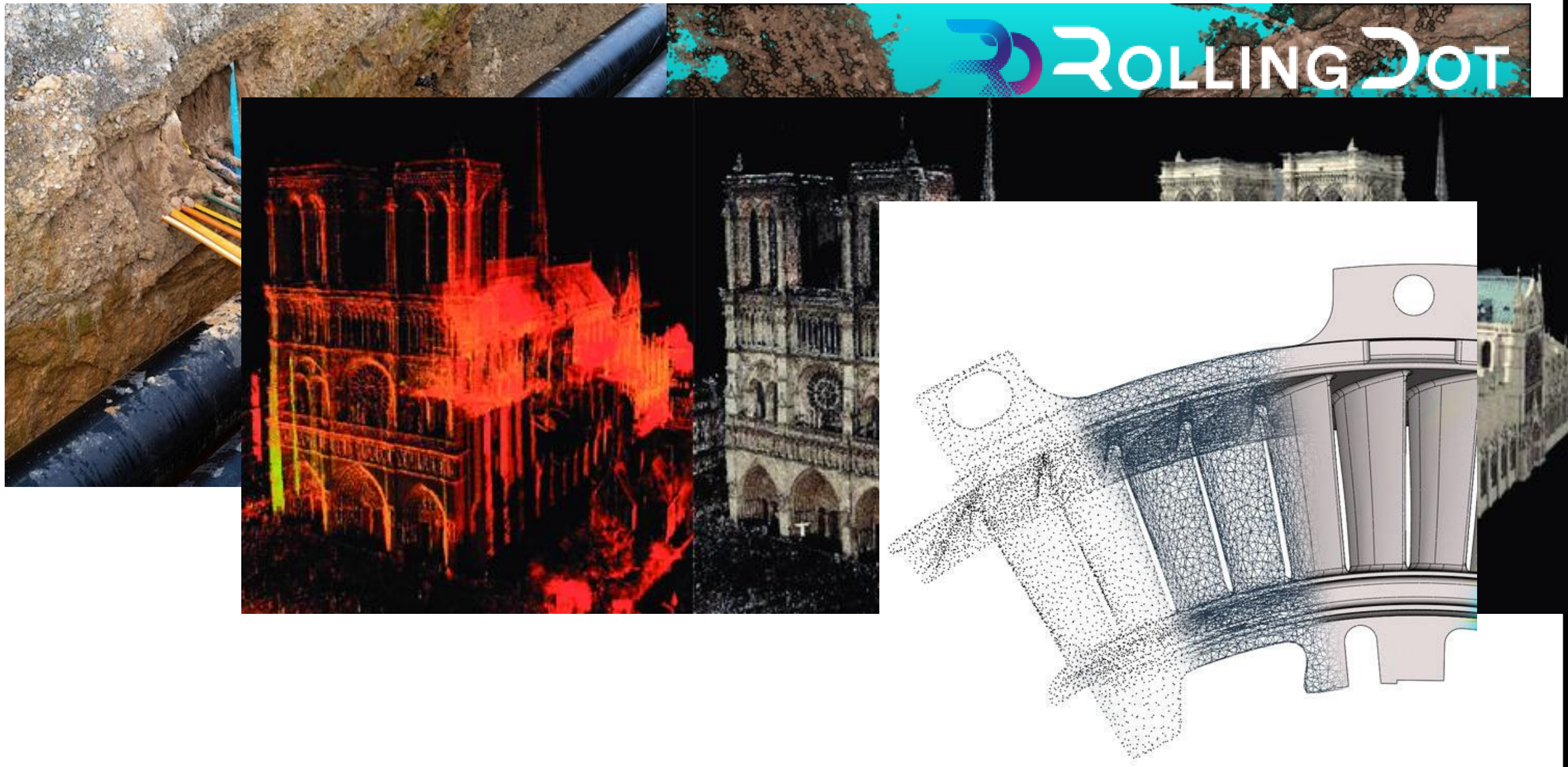
- Maintenance
- Cultural Heritage
- Reverse Engineering



3D point clouds – Applications

- Maintenance
- Cultural Heritage
- Reverse Engineering
- ...

- Toward 3D digital twins



3D point clouds – Acquisition processes

- Depth map (RGBD)

3D point clouds – Acquisition processes

- Depth map (RGBD)
- Photogrammetry

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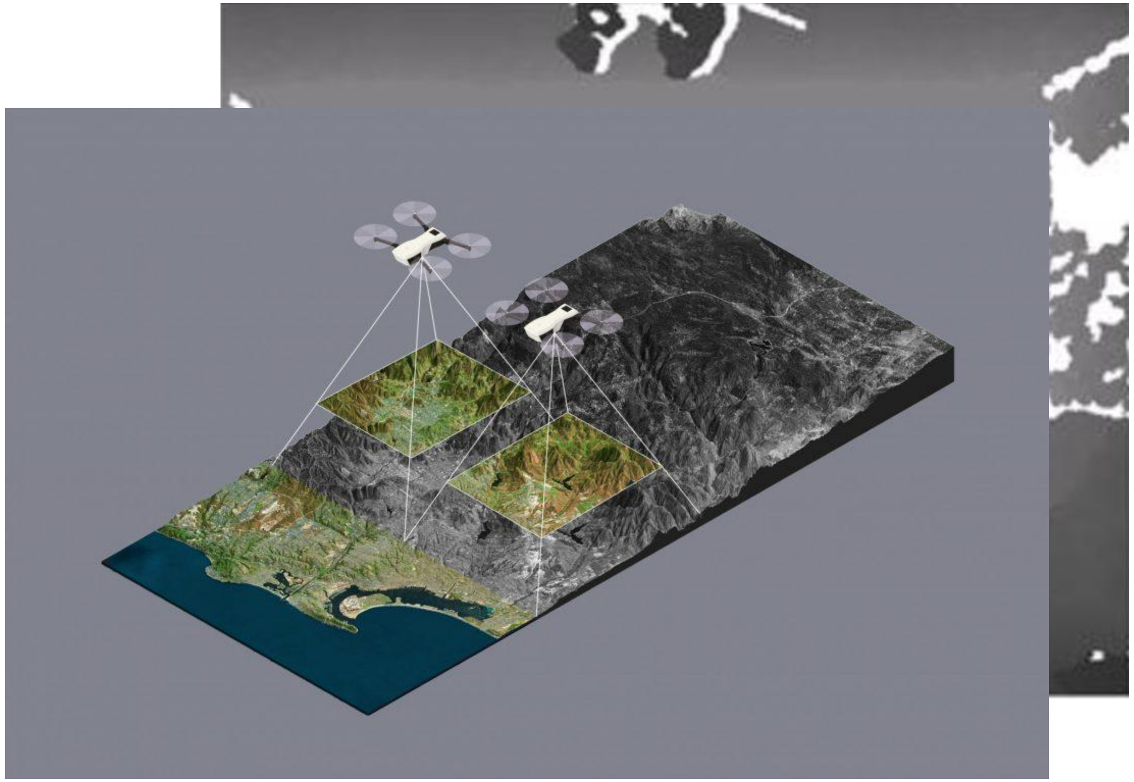
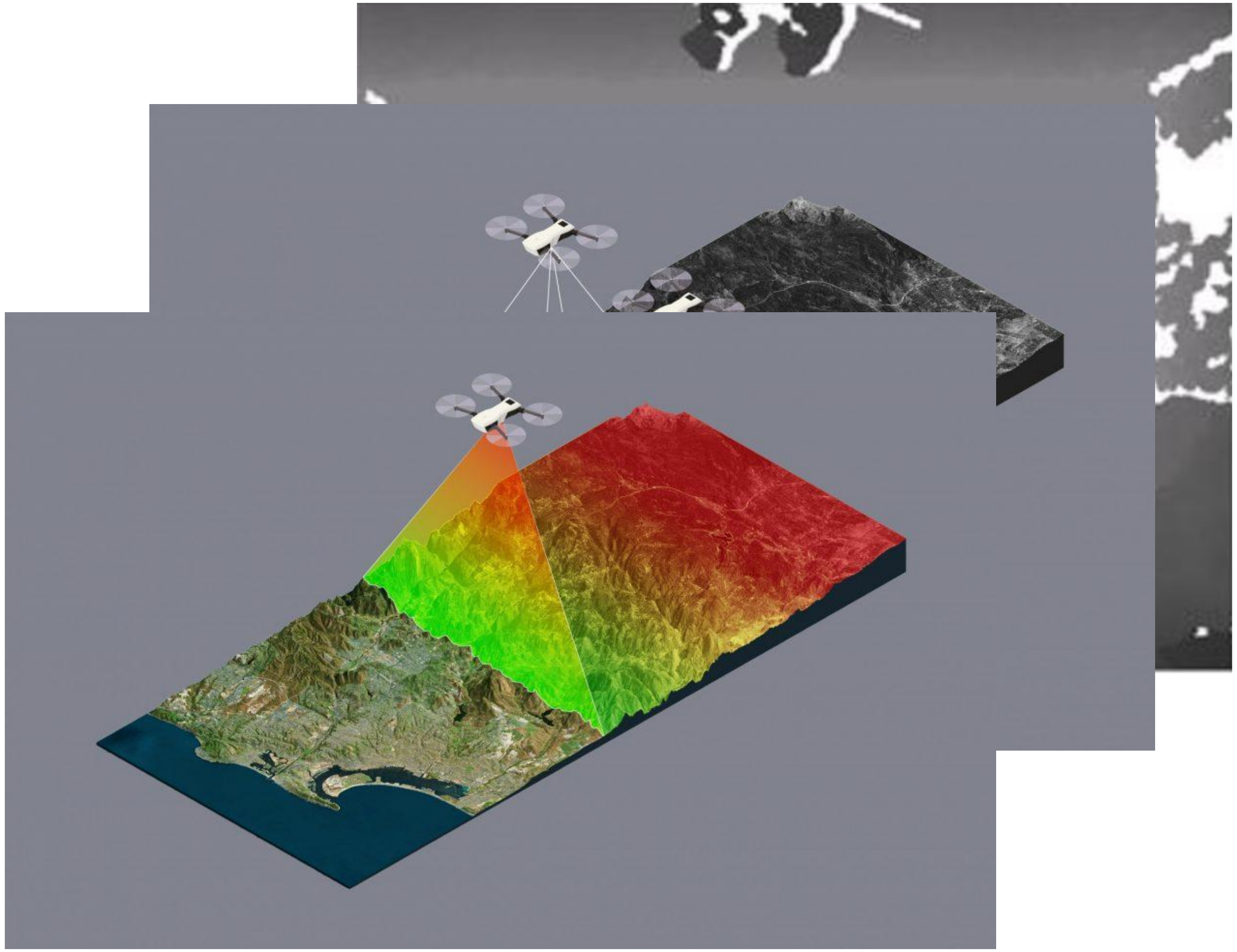


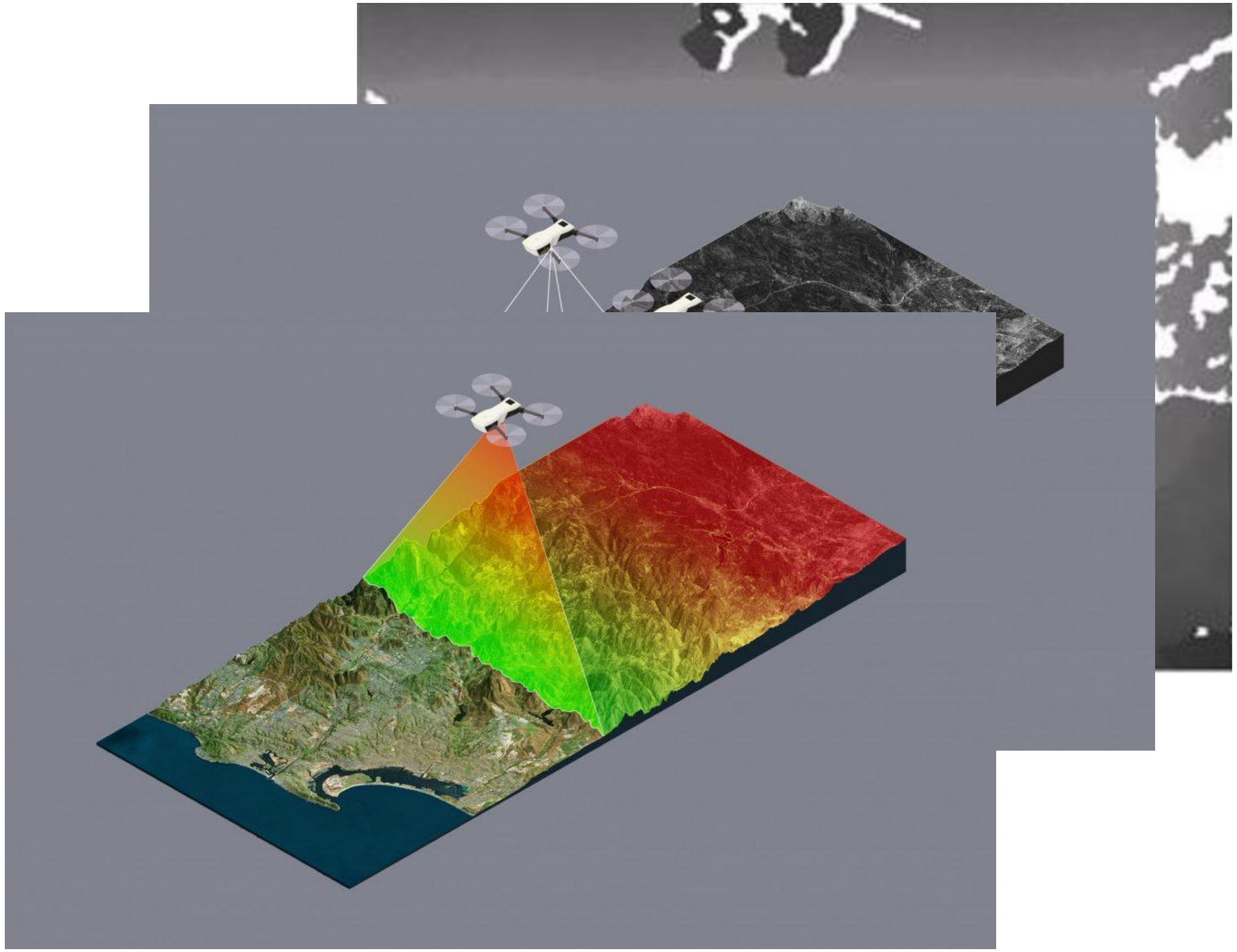
3D point clouds – Acquisition processes

- Depth map (RGBD)
- Photogrammetry
- LiDAR



3D point clouds – Acquisition processes

- Depth map (RGBD)
- Photogrammetry
- LiDAR
- …



3D point clouds – Acquisition processes

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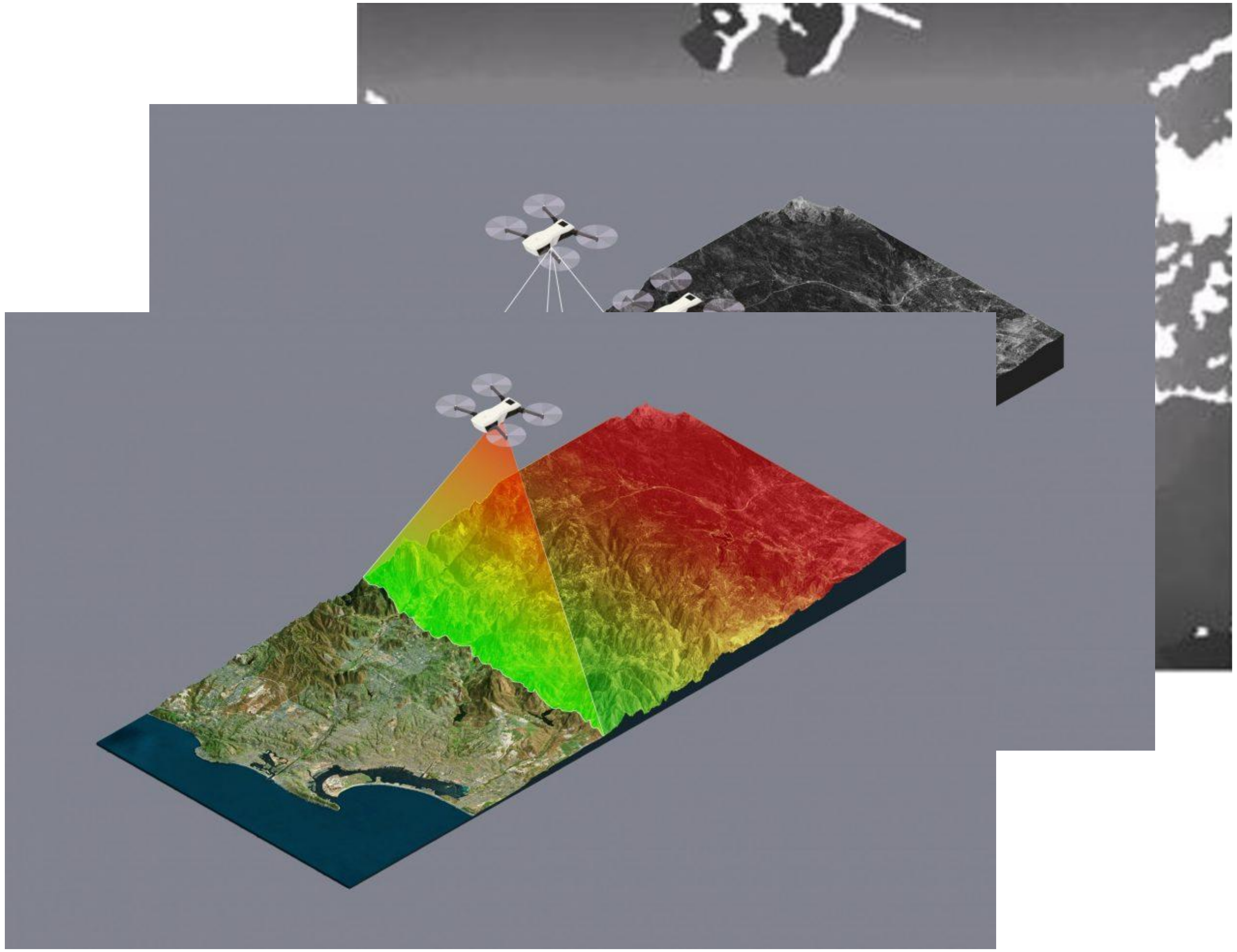
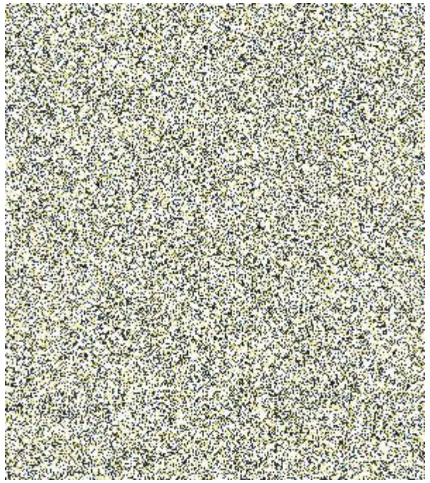


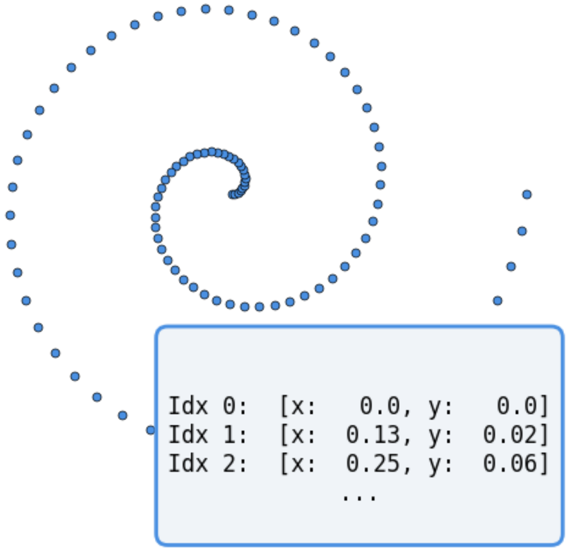
3D point clouds – Properties

- Unordered data

≠

permuted pixels





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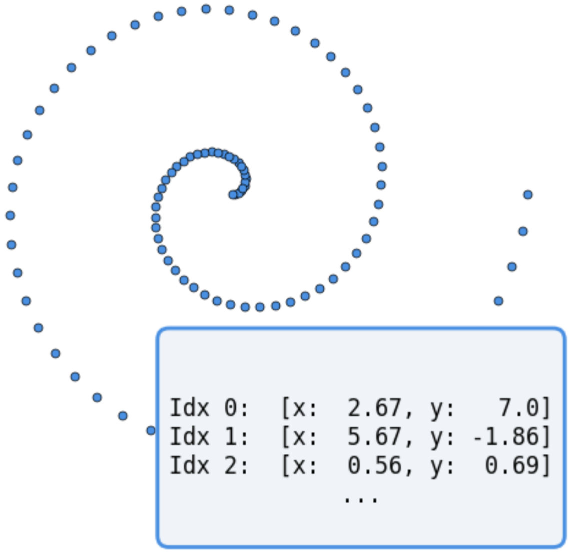
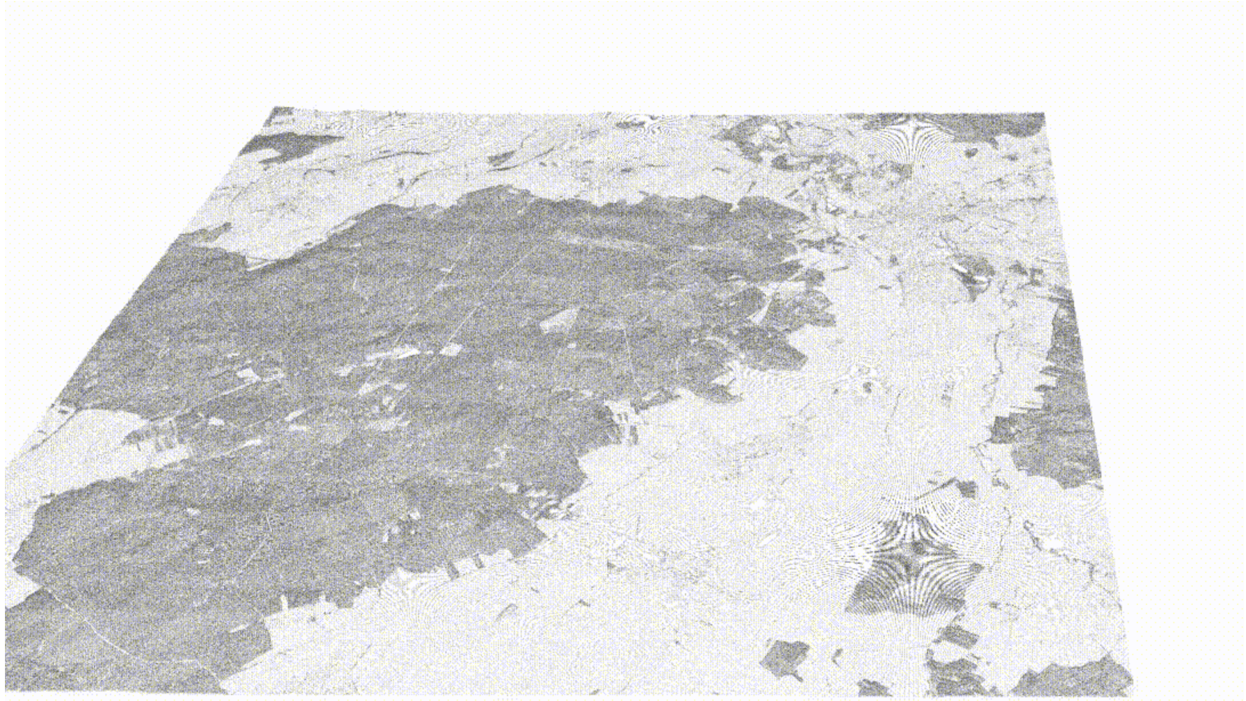
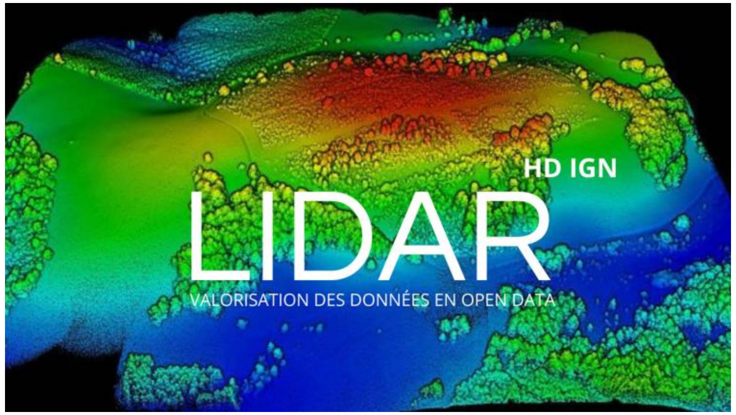


permuted points

3D point clouds – Properties

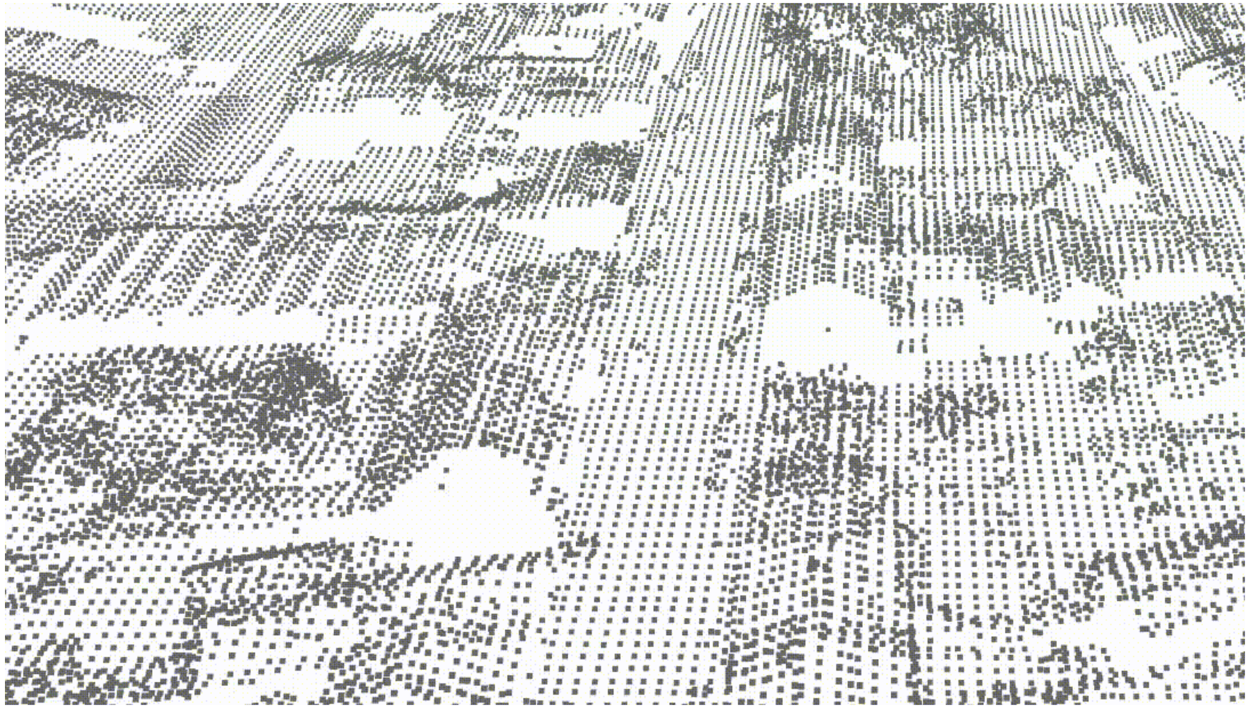
- Unordered data
- Geometric complexity

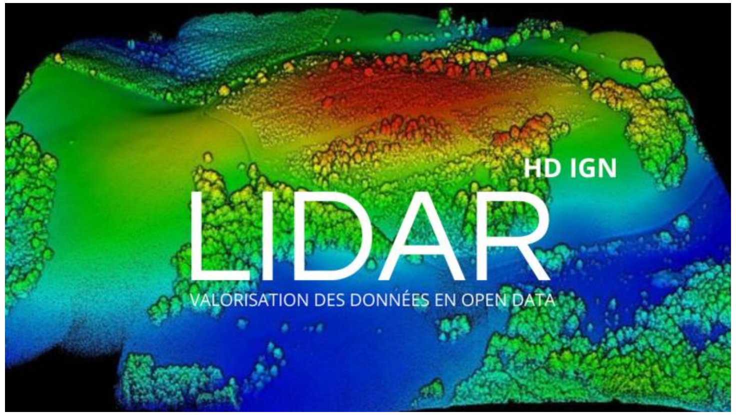




3D point clouds – Properties

- Unordered data
- Geometric complexity
- Artefacts
  - Sampling variation
  - Measurement noise and outliers
  - Registration errors
  - Missing data (occlusions)





3D point clouds – Acquired data

- 3D coordinates
- Sensor positions
- Attributes (depending on the acquisition process)
  - Colors
  - Intensity / Reflectivity
  - Normal vectors (not captured but can be estimated)
  - ...

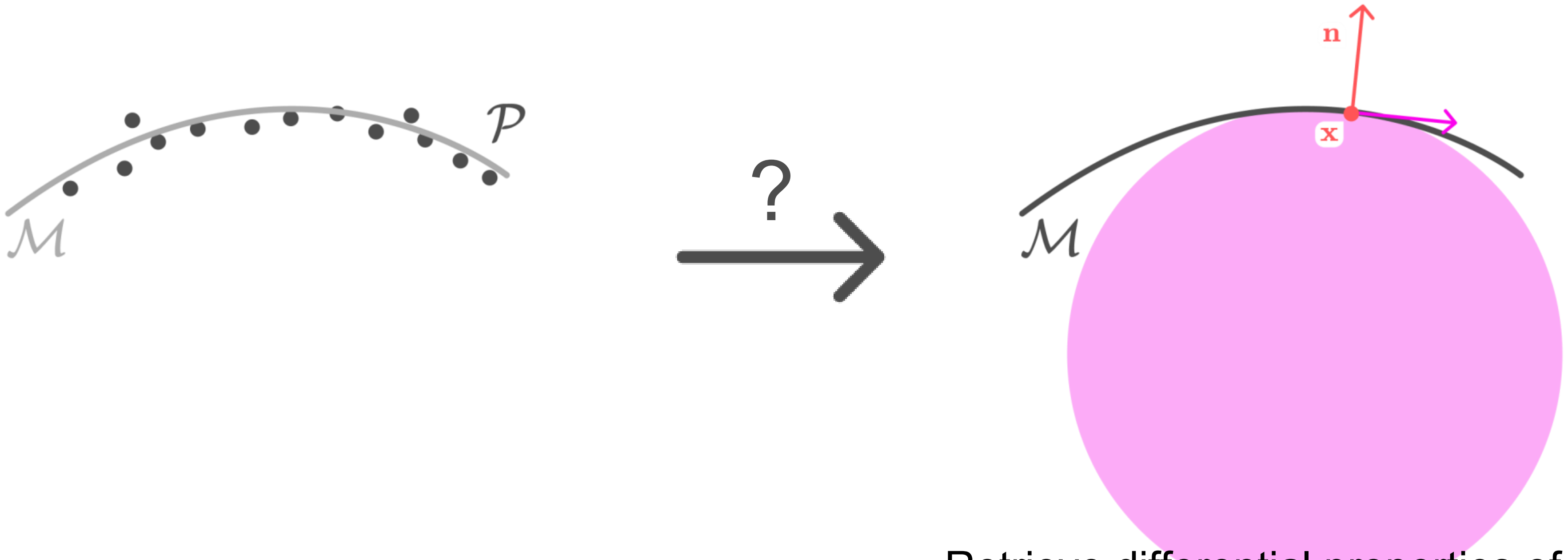
Context

Survey on differential estimators for 3d point clouds

Context

Survey on differential estimators for 3d point clouds

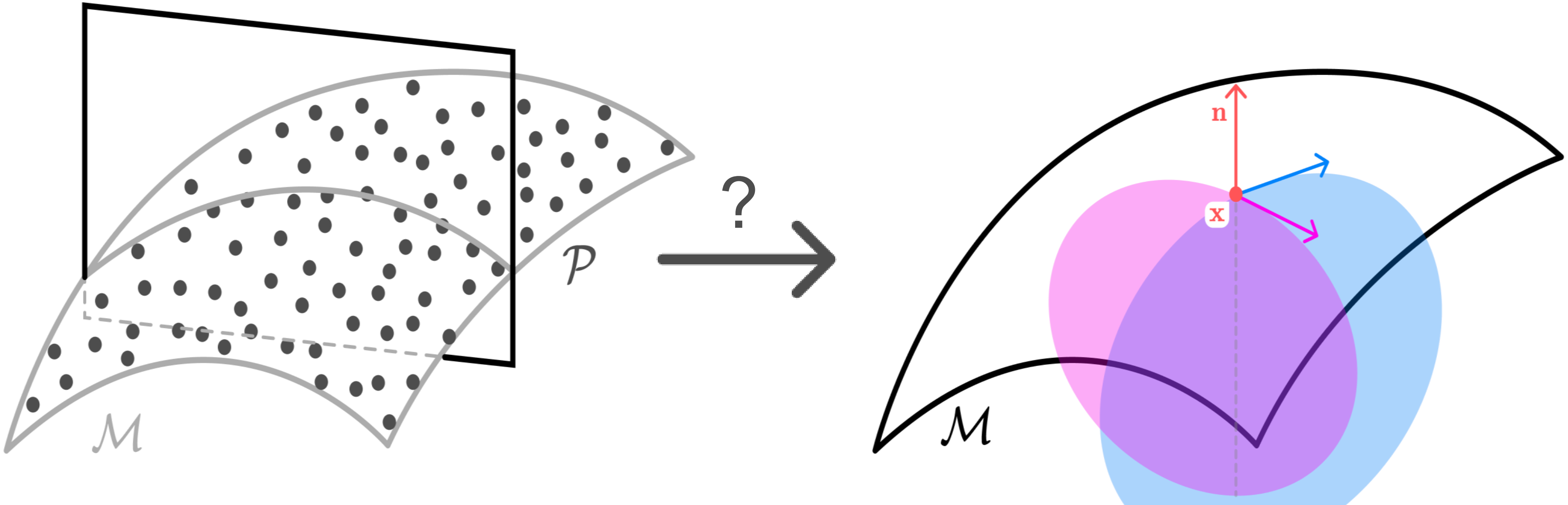
Differential



3D points sampling an unknown scanned smooth surface

Retrieve differential properties of the surface – ie. surface curvature

Differential



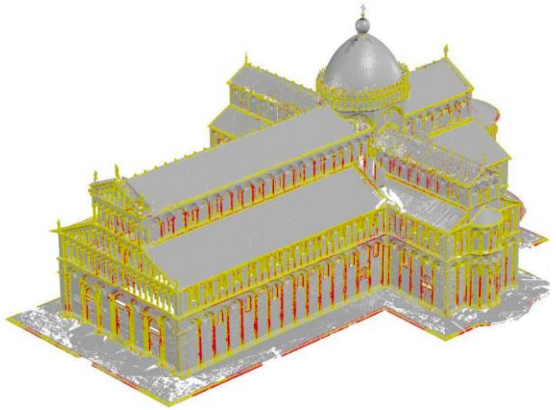
3D points sampling an unknown scanned smooth surface

Retrieve differential properties of the surface
– ie. surface curvature

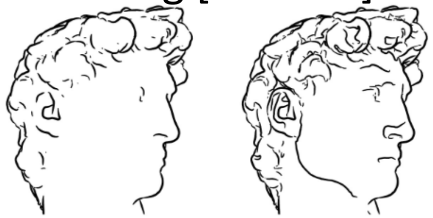
Differential – Usage Example

- Registration [GMG*05]

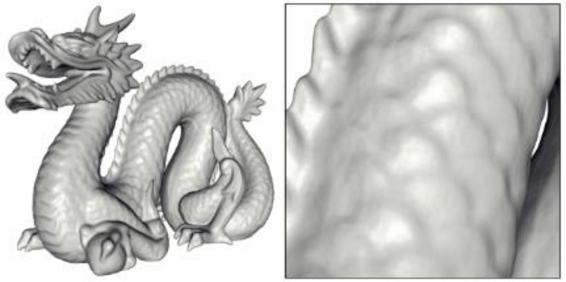
- Classification [HLP*21]



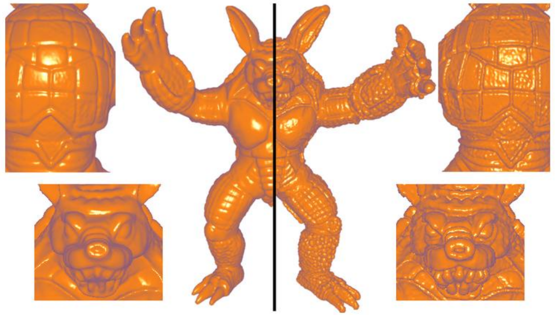
- Rendering [DFR*03]



- Meshing [KBH06]

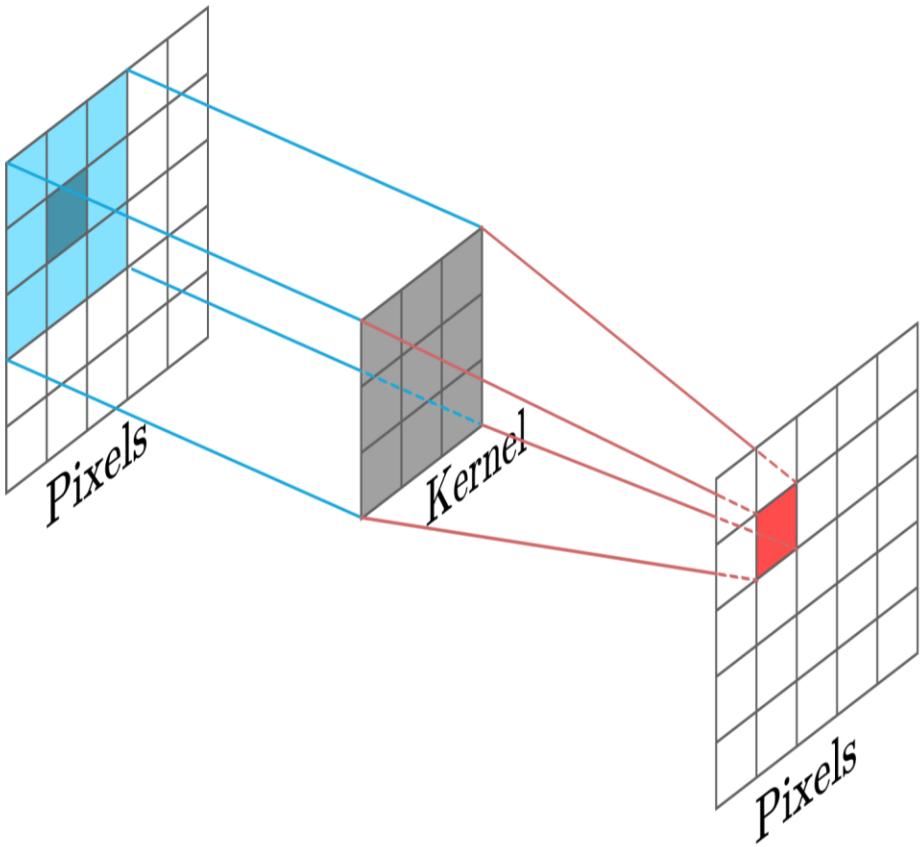


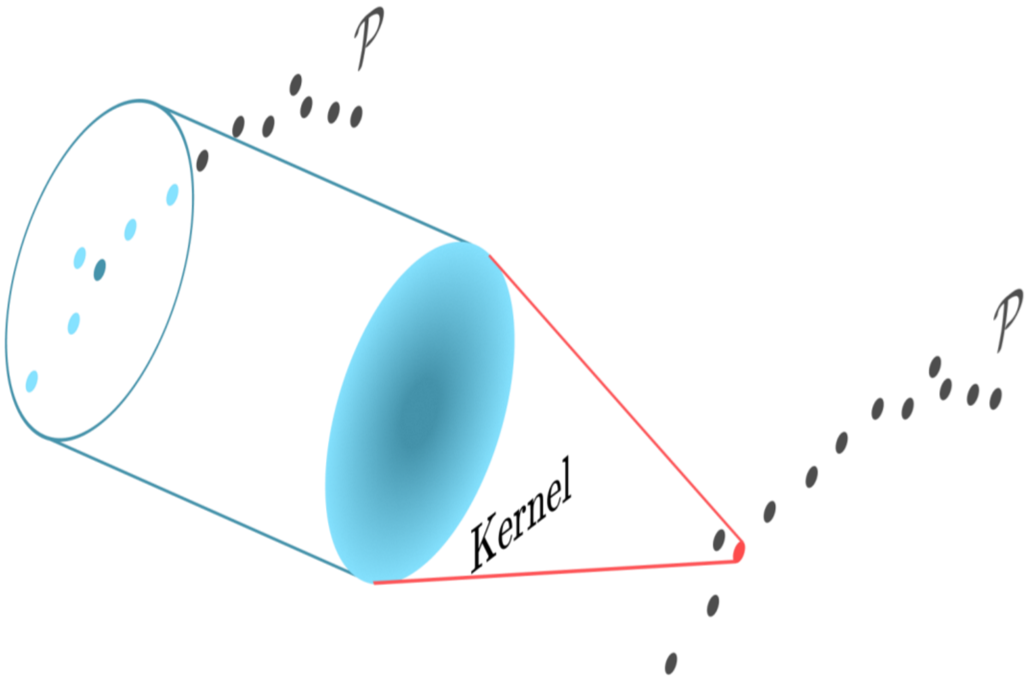
- Shape editing [BDC18]



Differential – Intuitions

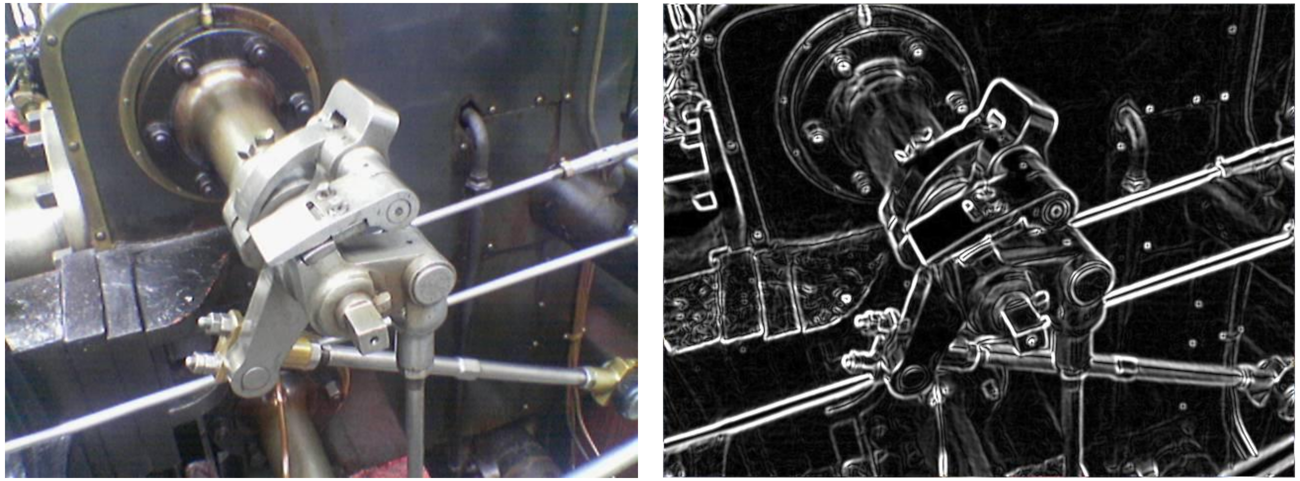
- Image analogy: convolution and scale



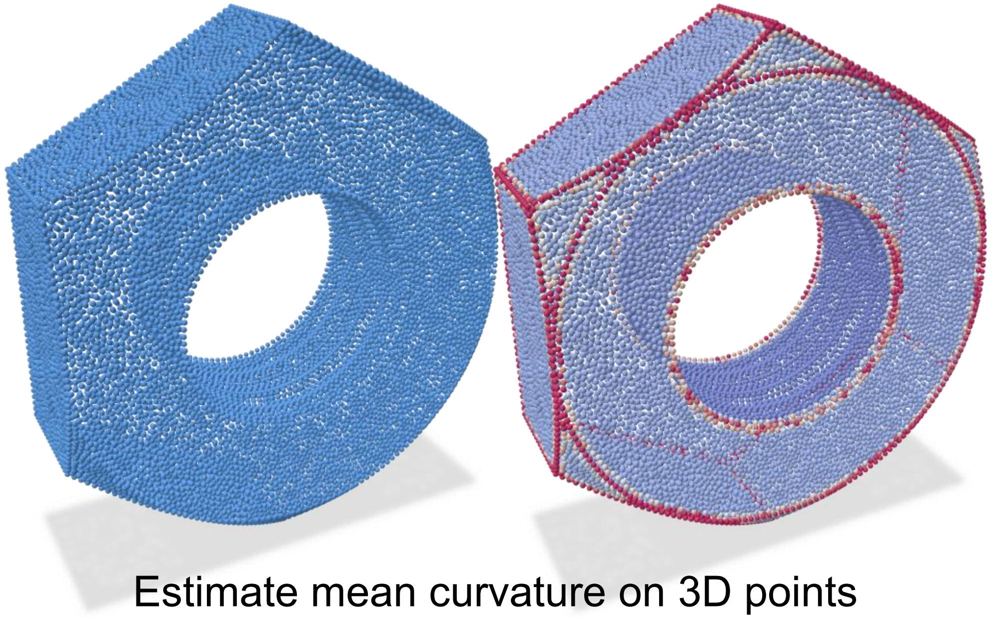


Differential – Intuitions

- Example of contour detection



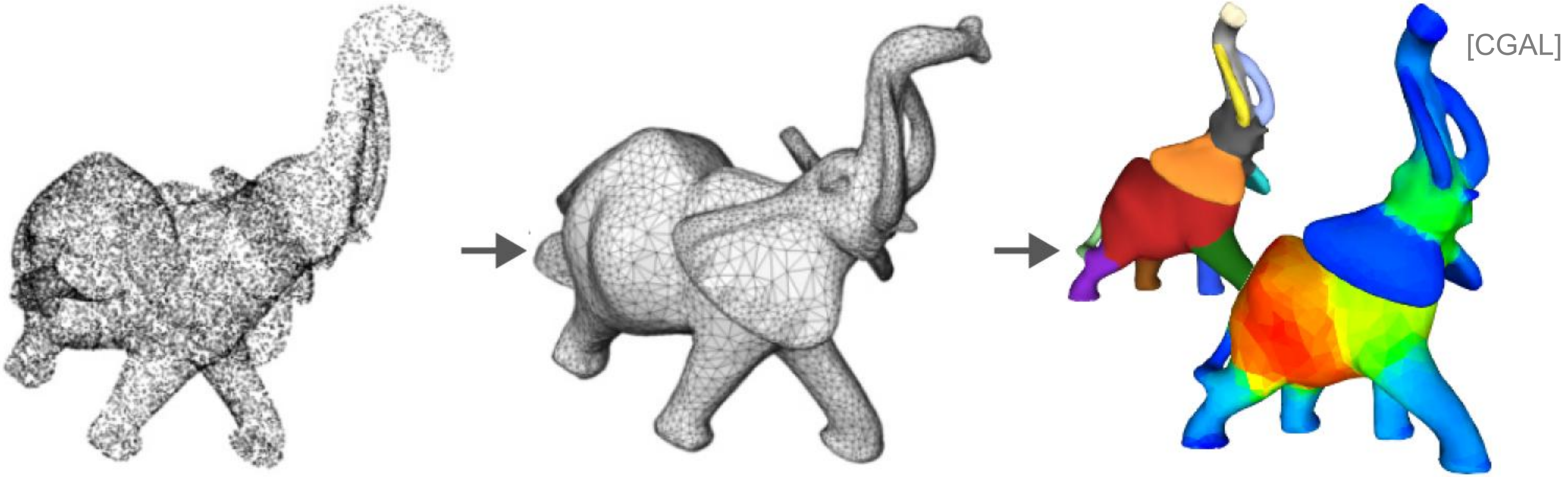
Sobel filter to compute 2D image gradients



Estimate mean curvature on 3D points

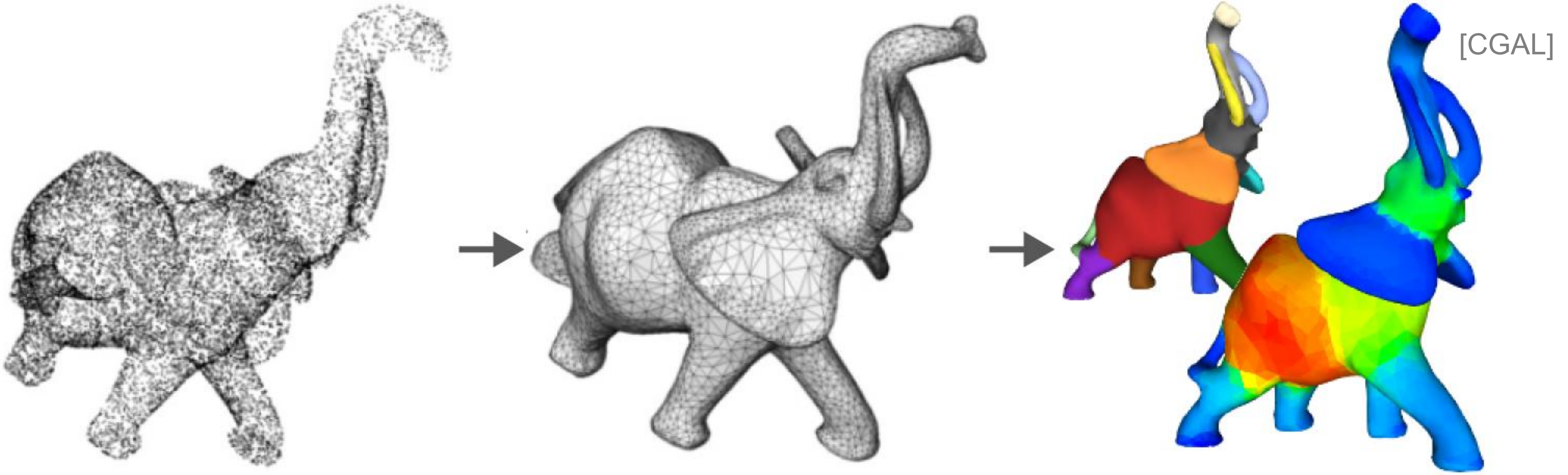
$$\mathbf{G}_x = \begin{bmatrix} -1 & 0 & +1 \\ -2 & 0 & +2 \\ -1 & 0 & +1 \end{bmatrix} * \mathbf{A} \qquad \mathbf{G}_y = \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ +1 & +2 & +1 \end{bmatrix} * \mathbf{A} \qquad \mathbf{G} = \sqrt{{\mathbf{G}_x}^2 + {\mathbf{G}_y}^2}$$

Differential – Mesh-based process



point cloud mesh processing

Differential – Mesh-based process

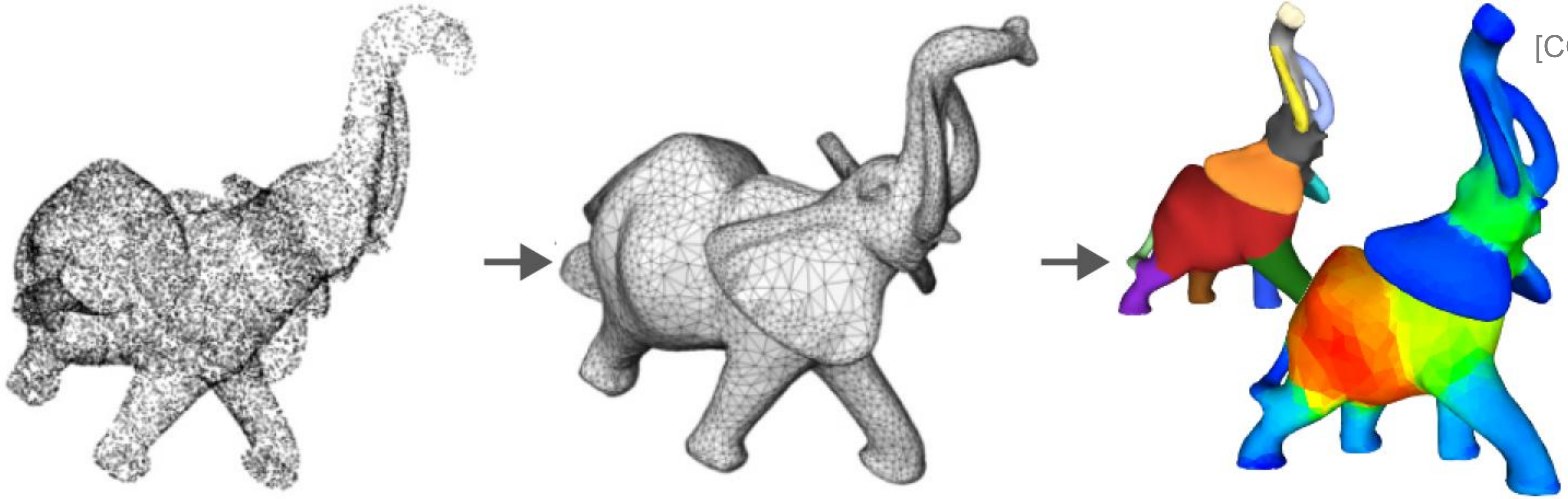


point cloud | mesh | processing

A survey of **surface reconstruction** from point clouds [BTS*17]

Mesh Statistics for Robust **Curvature Estimation** [VVP*16]

Differential – Mesh-based process



[CGAL]

point cloud mesh processing

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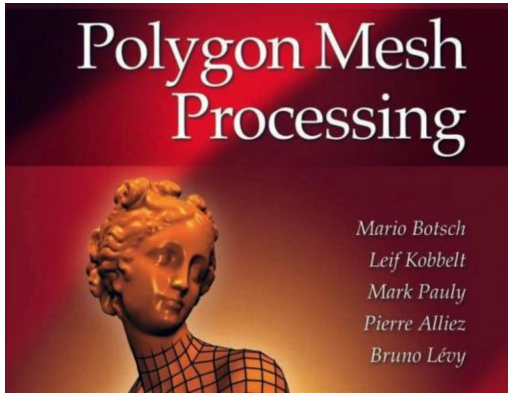
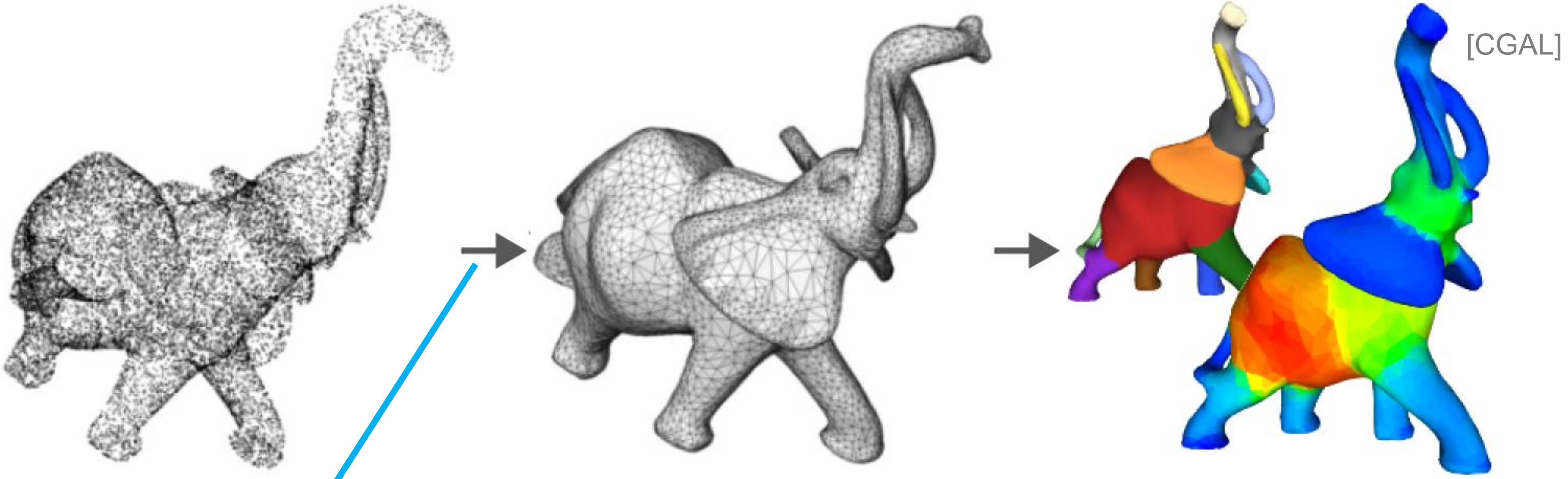
Mesh Statistics for Robust **Curvature Estimation** [VVP*16]



Differential – Mesh-based process



point cloud

mesh

processing

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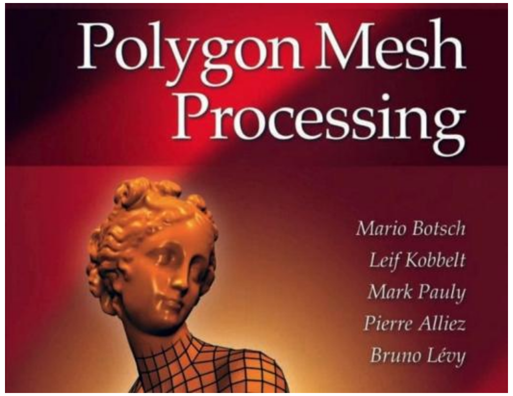
Mesh Statistics for Robust **Curvature Estimation** [VVP*16]



working on raw data

Context – Scope

- Differential properties estimation in a raw 3D point cloud
- Special focus on mathematical models
  - with theoretical guarantees
  - their use in Machine Learning models
- Unified benchmark

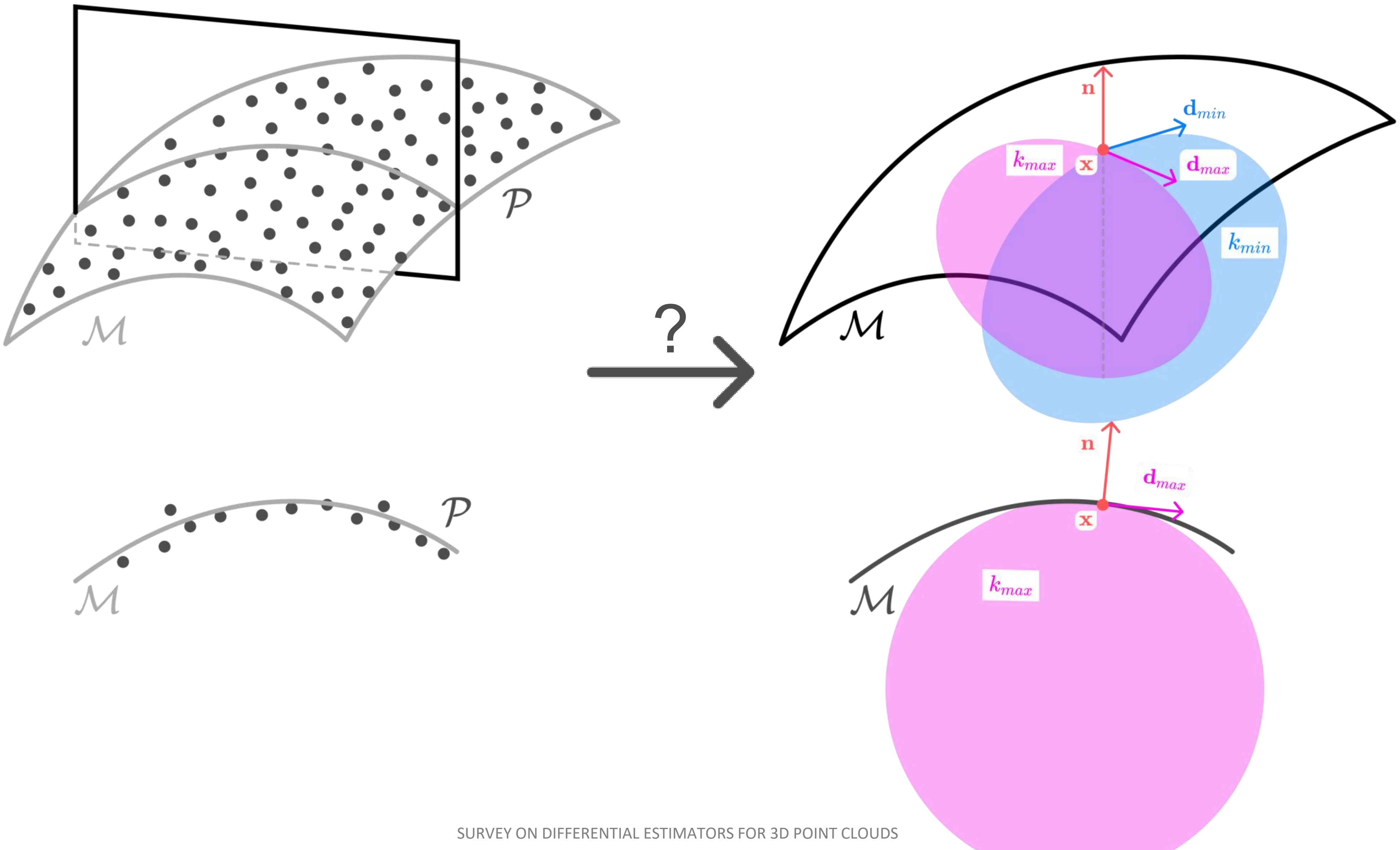
PLAN

1. Context
2. Fundamentals
3. Methods
4. Benchmark
5. Future works
6. Conclusion

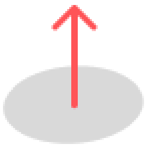
Fundamentals

02

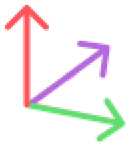
Fundamentals



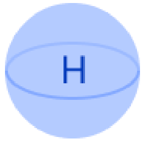
Fundamentals



Normal



Tangent frame

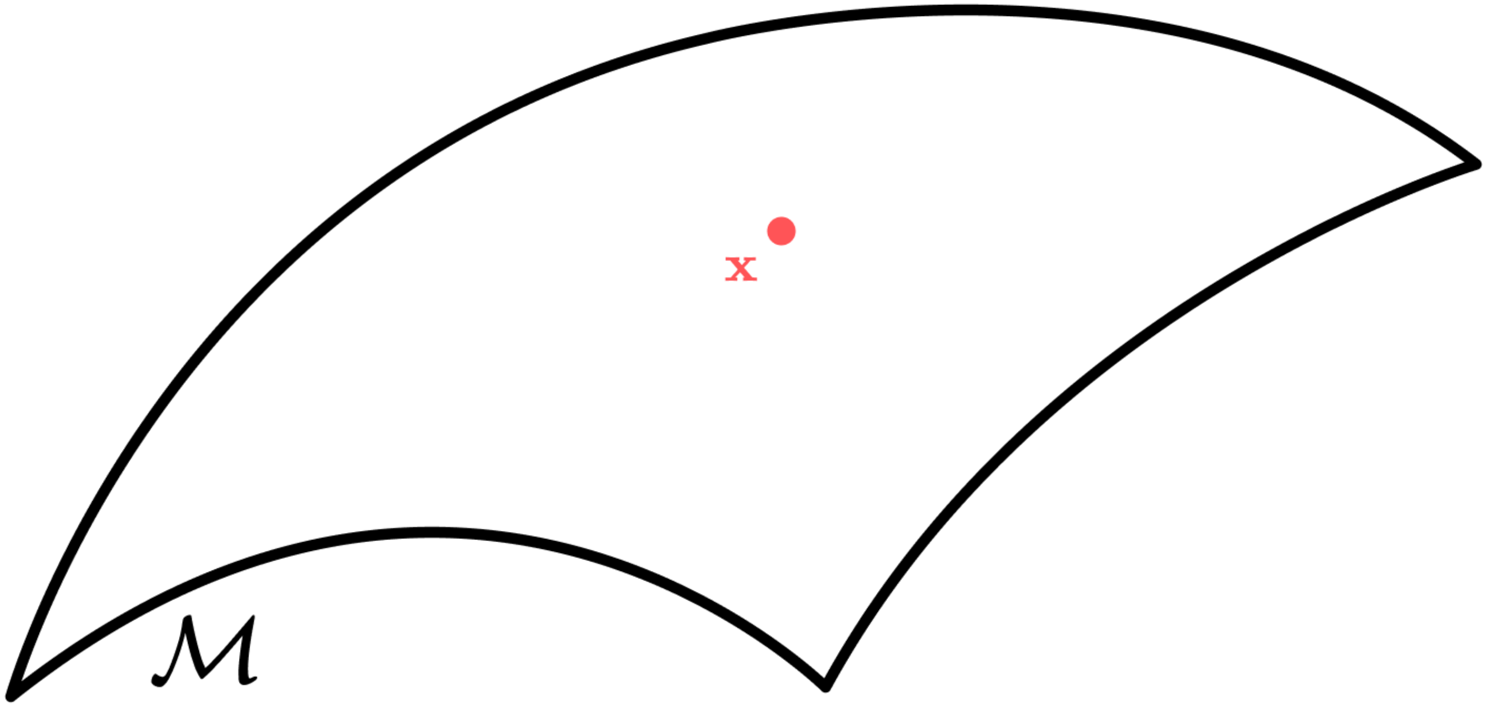


Mean curvature

Weingarten map
(principal curvature values and directions)

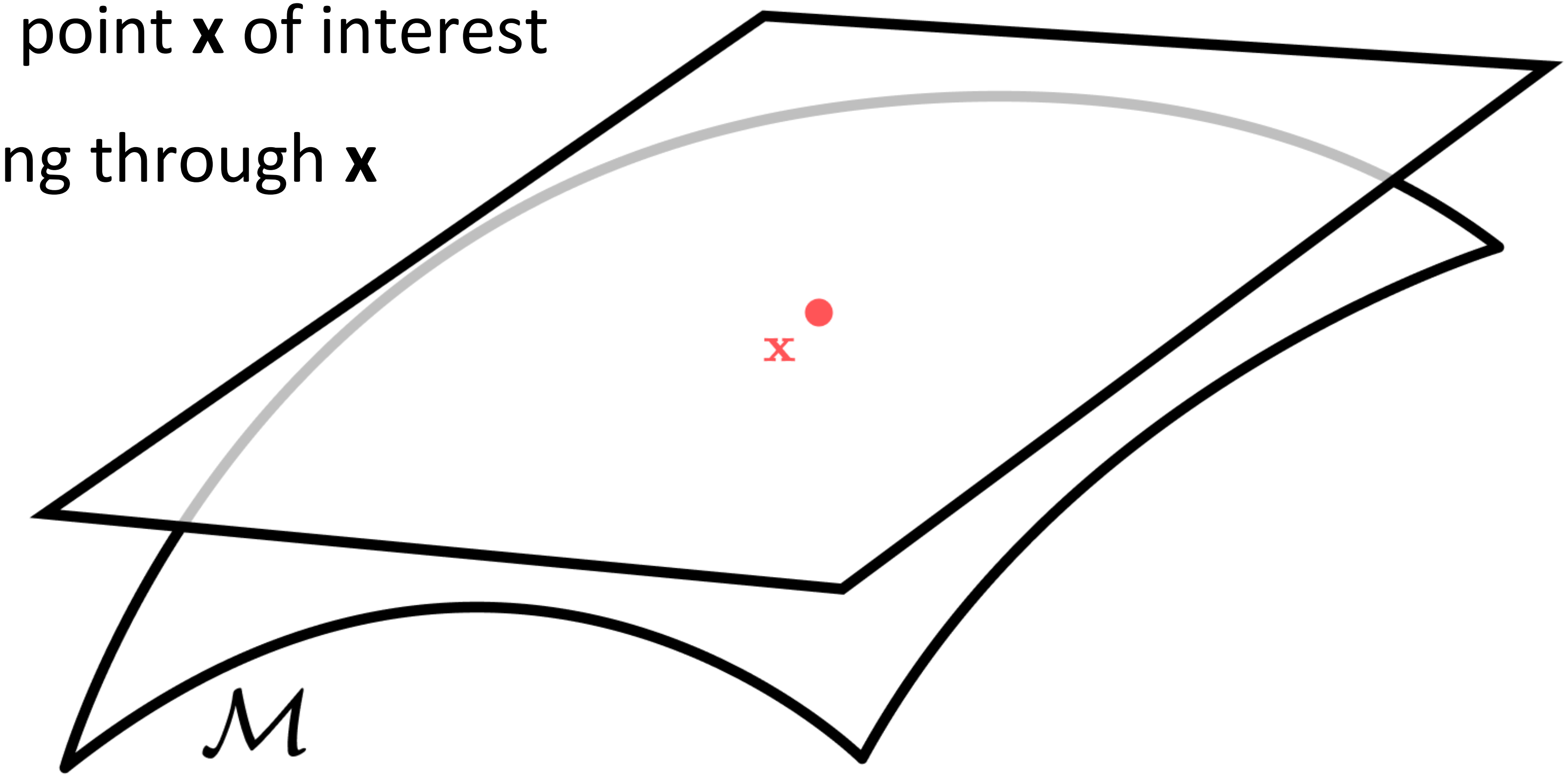
Normal vector

- Given a surface M and a point **x** of interest



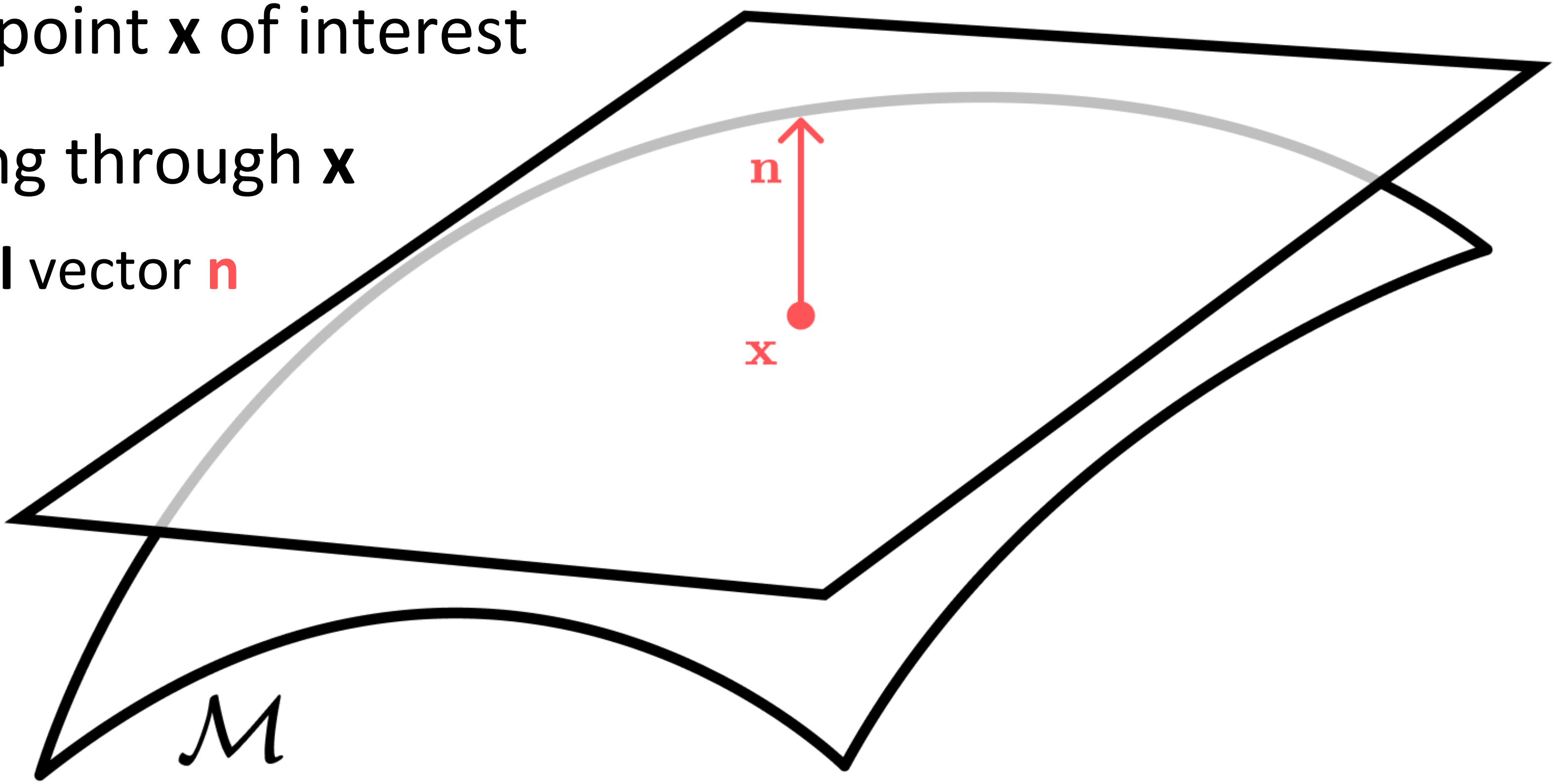
Normal vector

- Given a surface M and a point **x** of interest
- The **tangent plane** passing through **x**



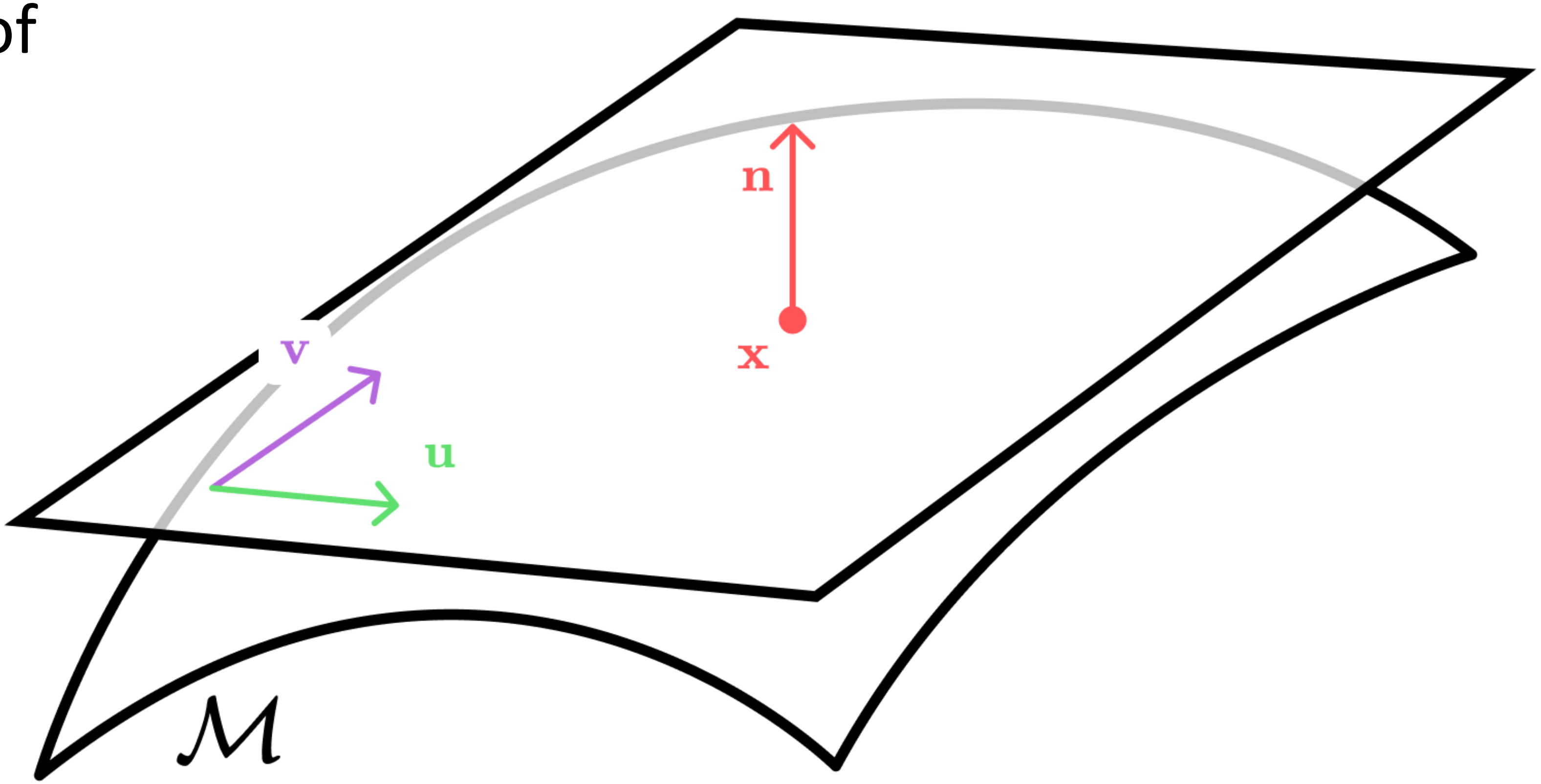
Normal vector

- Given a surface M and a point **x** of interest
- The **tangent plane** passing through **x**
  - Orthogonal to the **normal** vector **n**

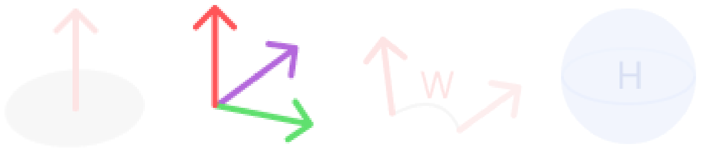


Tangent frame

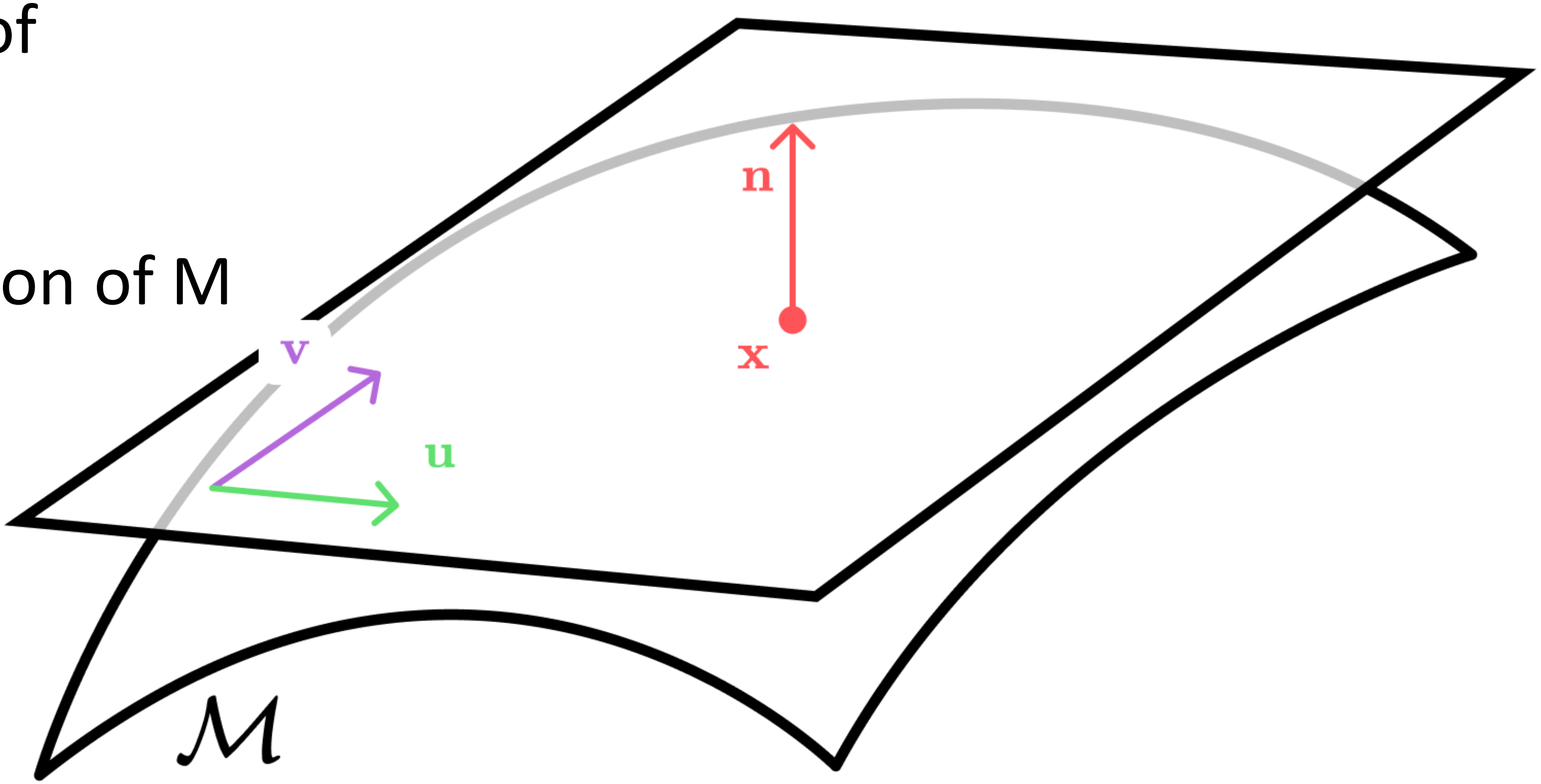
- **u**, **v** an orthogonal base of the **tangent frame**



Tangent frame

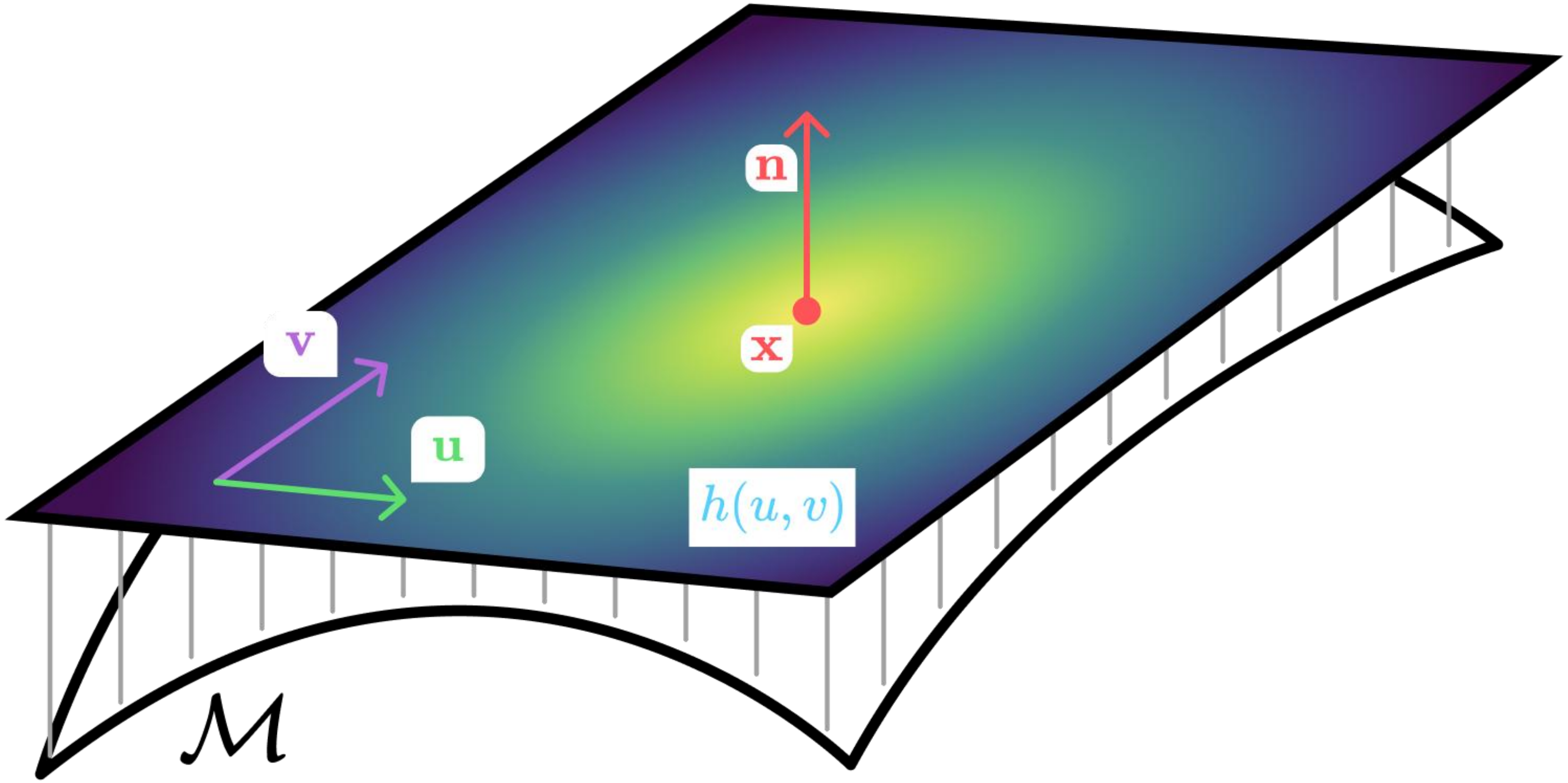


- **u**, **v** an orthogonal base of the **tangent frame**
- Let's find a parametrisation of M



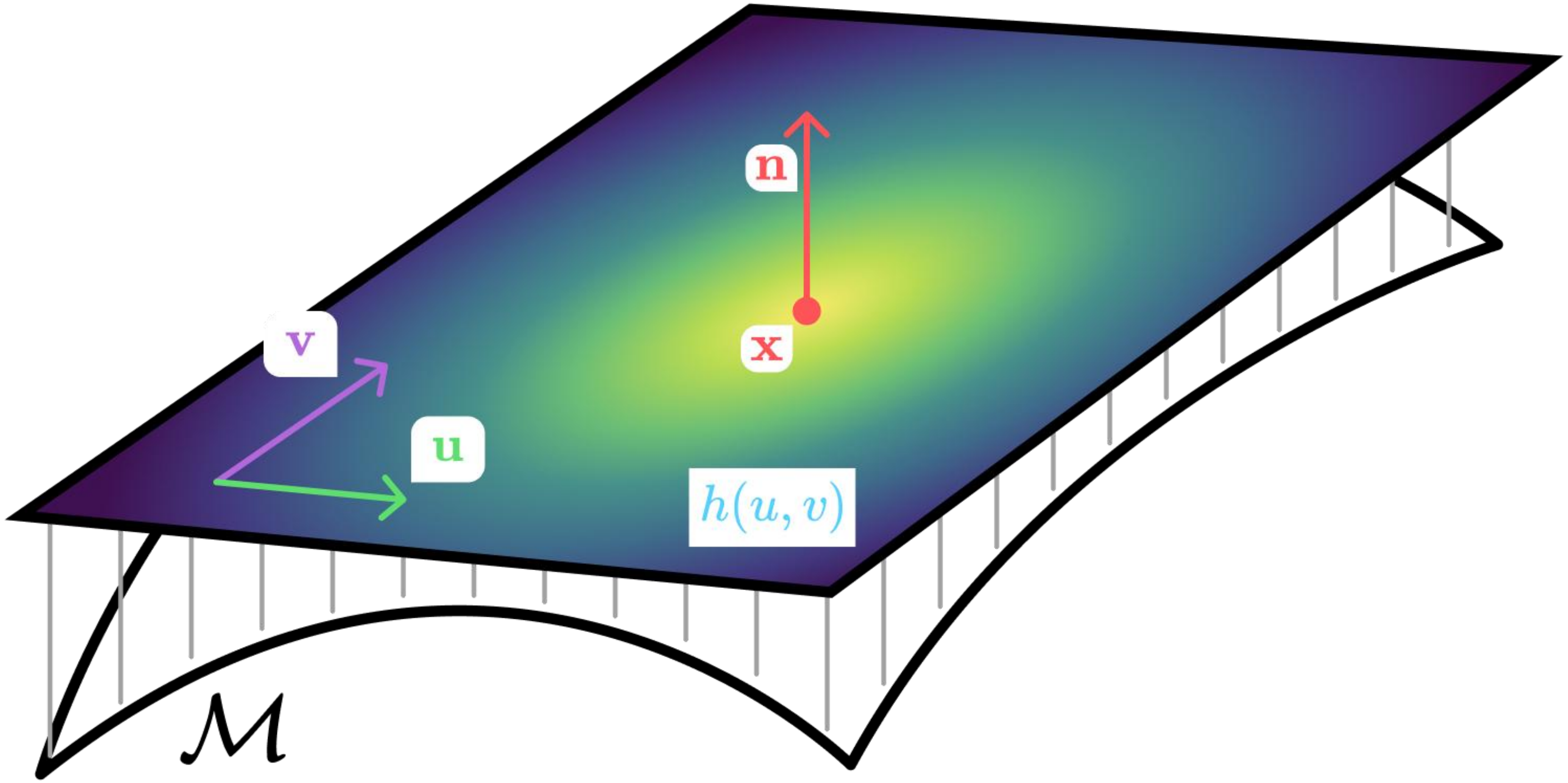
Tangent frame - parametrisation

- h(u,v) the **height field**



Tangent frame - parametrisation

- h(u,v) the **height field**
- Note **f(u,v,h(u,v))** the **monge patch** of M

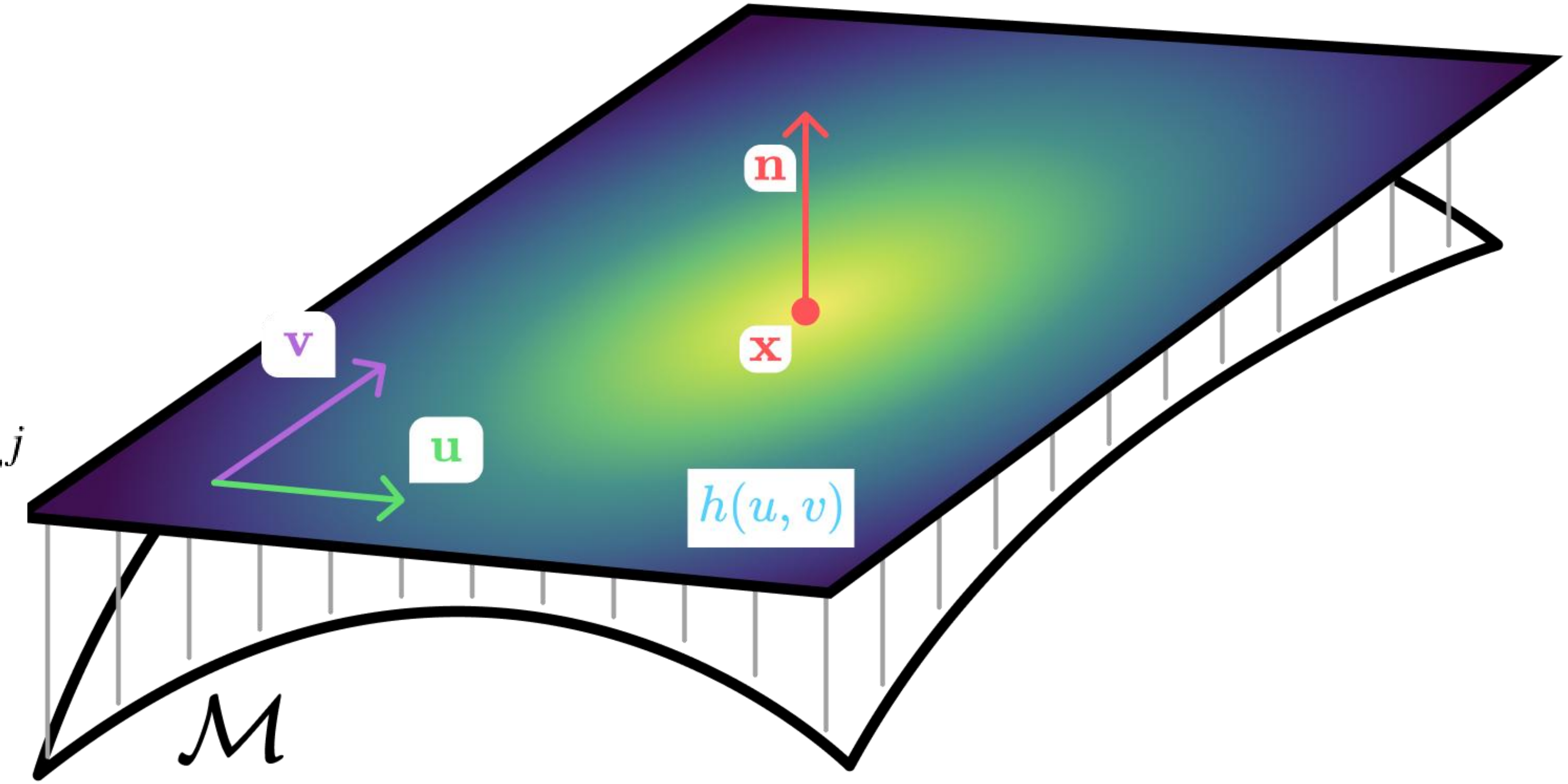


Tangent frame - parametrisation

- h(u,v) the **height field**
- Note **f(u,v,h(u,v))**

  the **monge patch** of M

$$h(u,v) = \sum_{k=0}^{\infty} \sum_{j=0}^{k} \frac{h_{u^{k-j}v^{j}}(0,0)}{(k-j)!\,j!} u^{k-j} v^{j}$$



Tangent frame – Fundamental forms

- The first fundamental form
  - **Intrinsic** properties of the surface

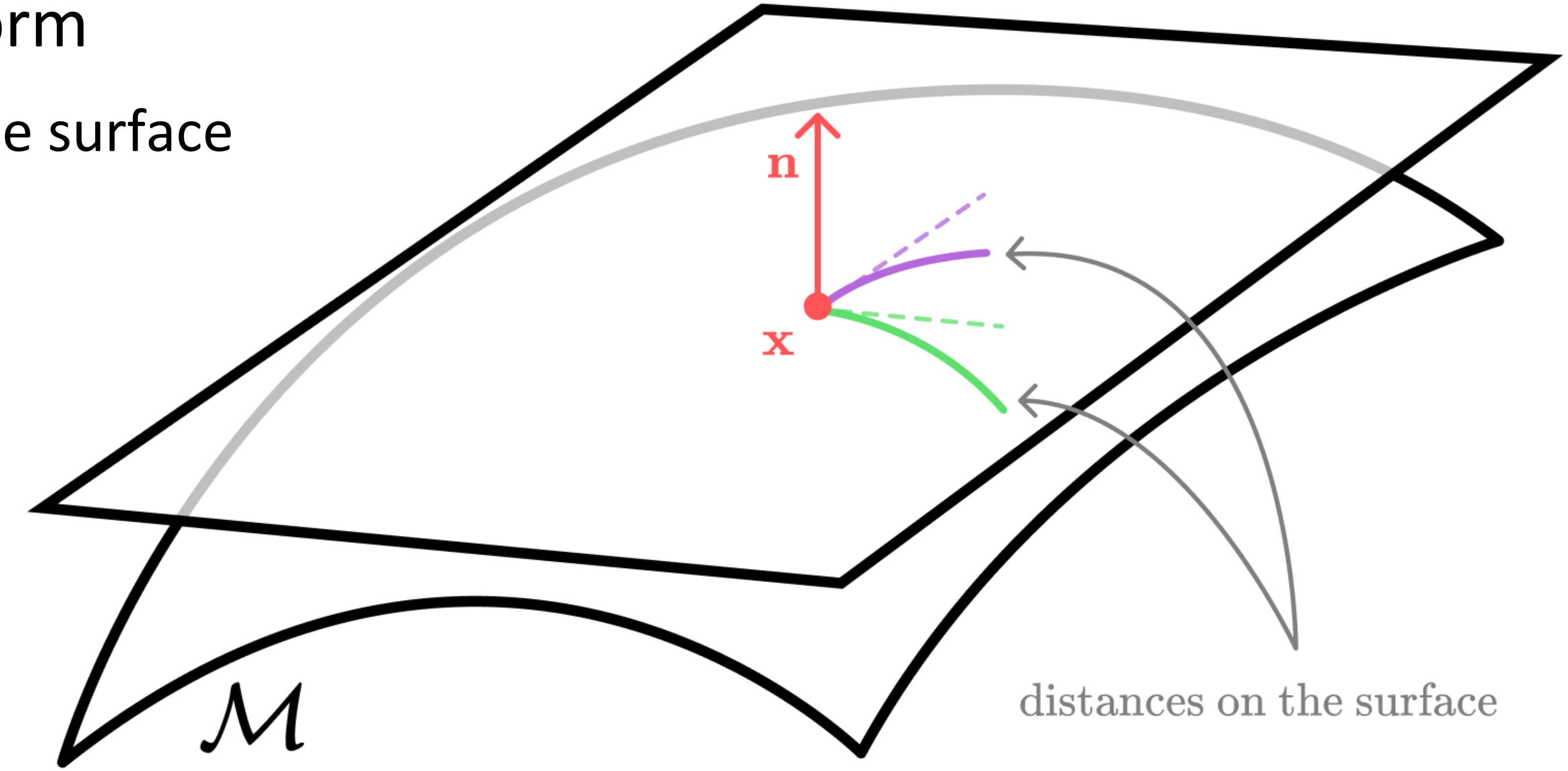
$$F_{\mathrm{I}} = \begin{pmatrix} E & F \\ F & G \end{pmatrix}$$

$$E = ||f_u||^2$$

$$F = f_u \cdot f_v$$

$$G = ||f_v||^2$$

(with $f_u = \frac{\partial f}{\partial u}$).



Tangent frame – Fundamental forms

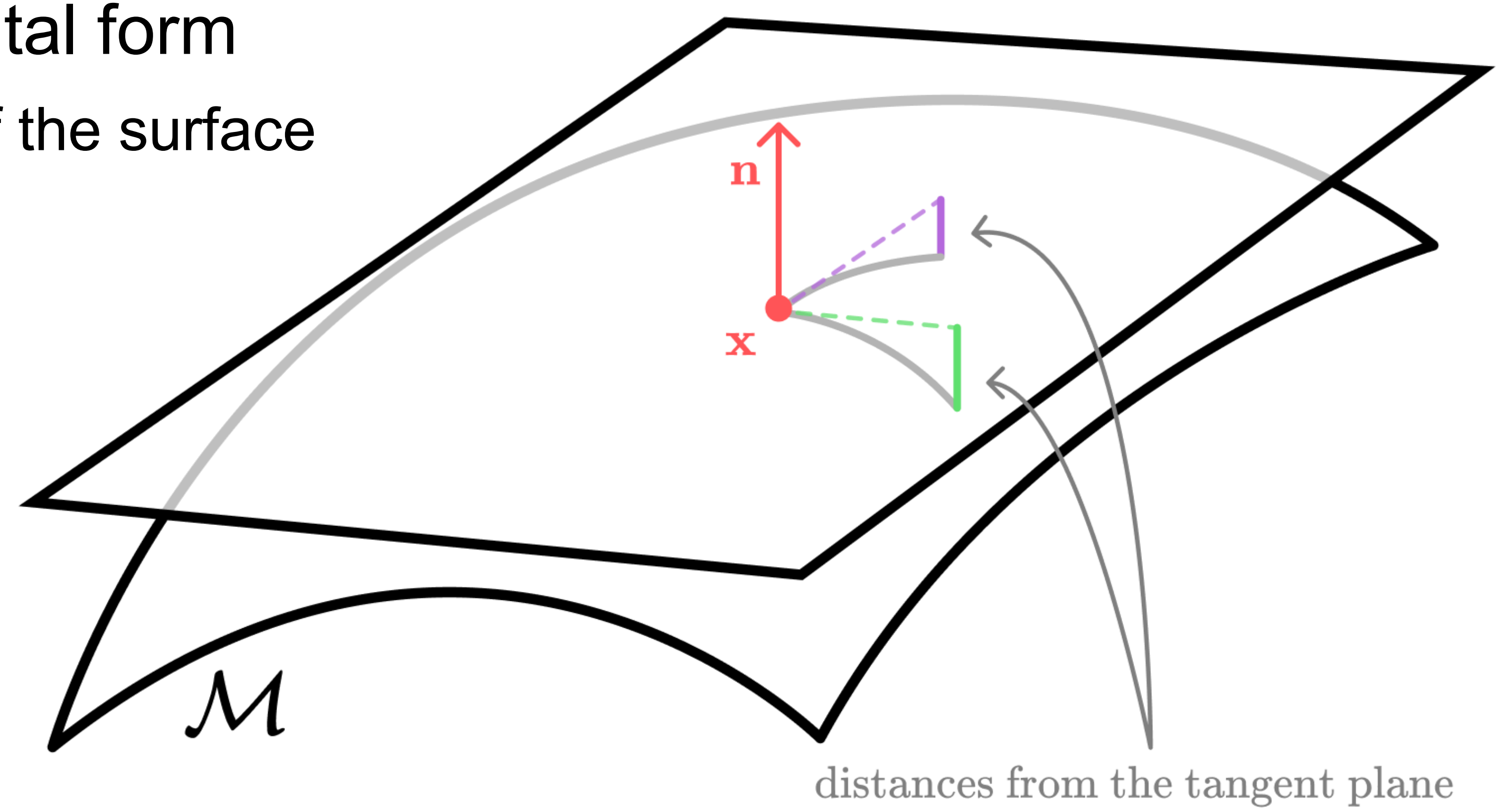
- The second fundamental form
  - **Extrinsic** properties of the surface

$$\mathrm{F_{II}} = \begin{pmatrix} L & M \\ M & N \end{pmatrix}$$

$$L = f_{uu} \cdot \mathbf{n}_f$$

$$M = f_{uv} \cdot \mathbf{n}_f$$

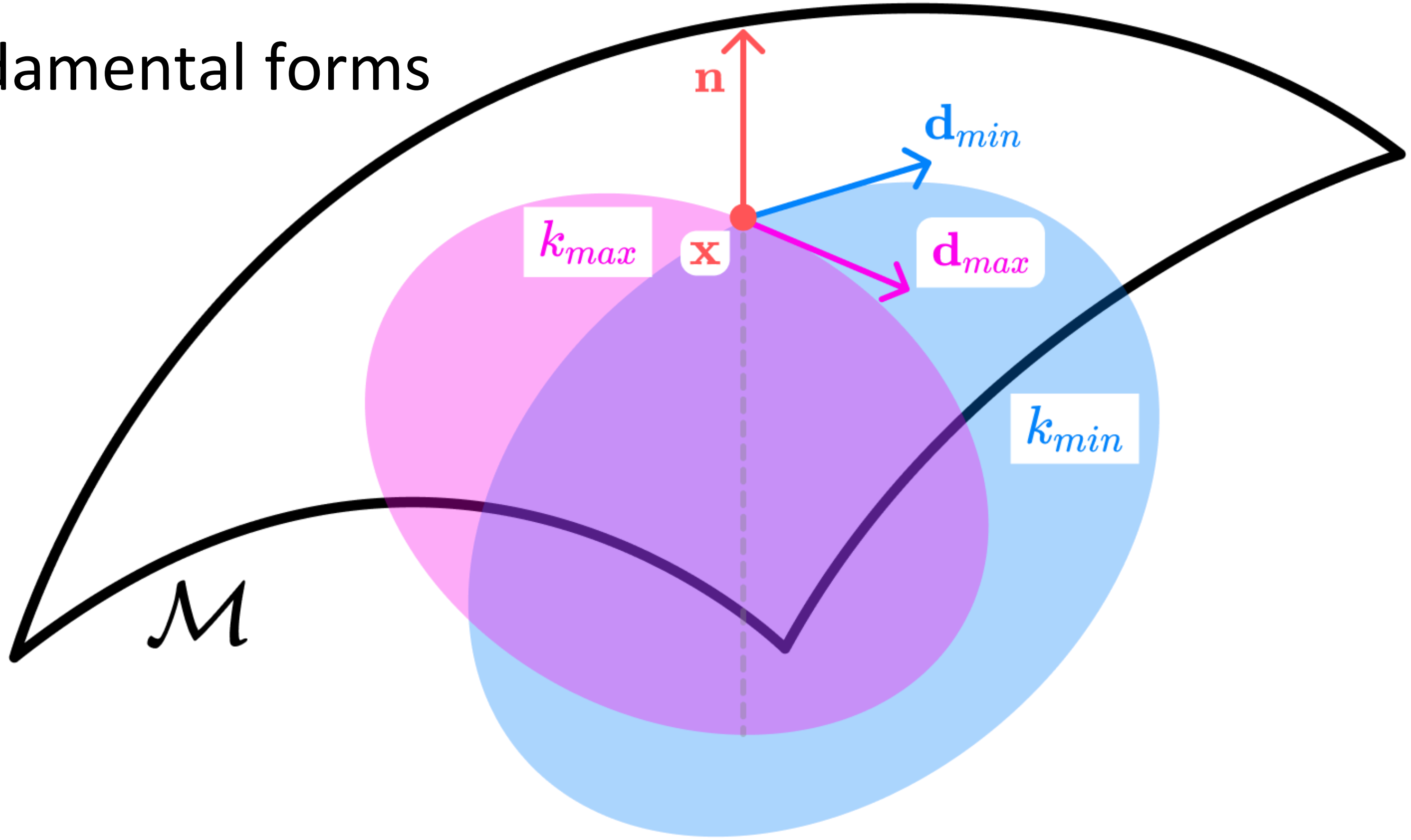
$$N = f_{vv} \cdot \mathbf{n}_f$$



Weingarten Map

- Also called **Shape Operator**
- **Multiplication** of the fundamental forms

$$\mathrm{W} = \mathrm{F}_{\mathrm{I}}^{-1}\mathrm{F}_{\mathrm{II}}$$



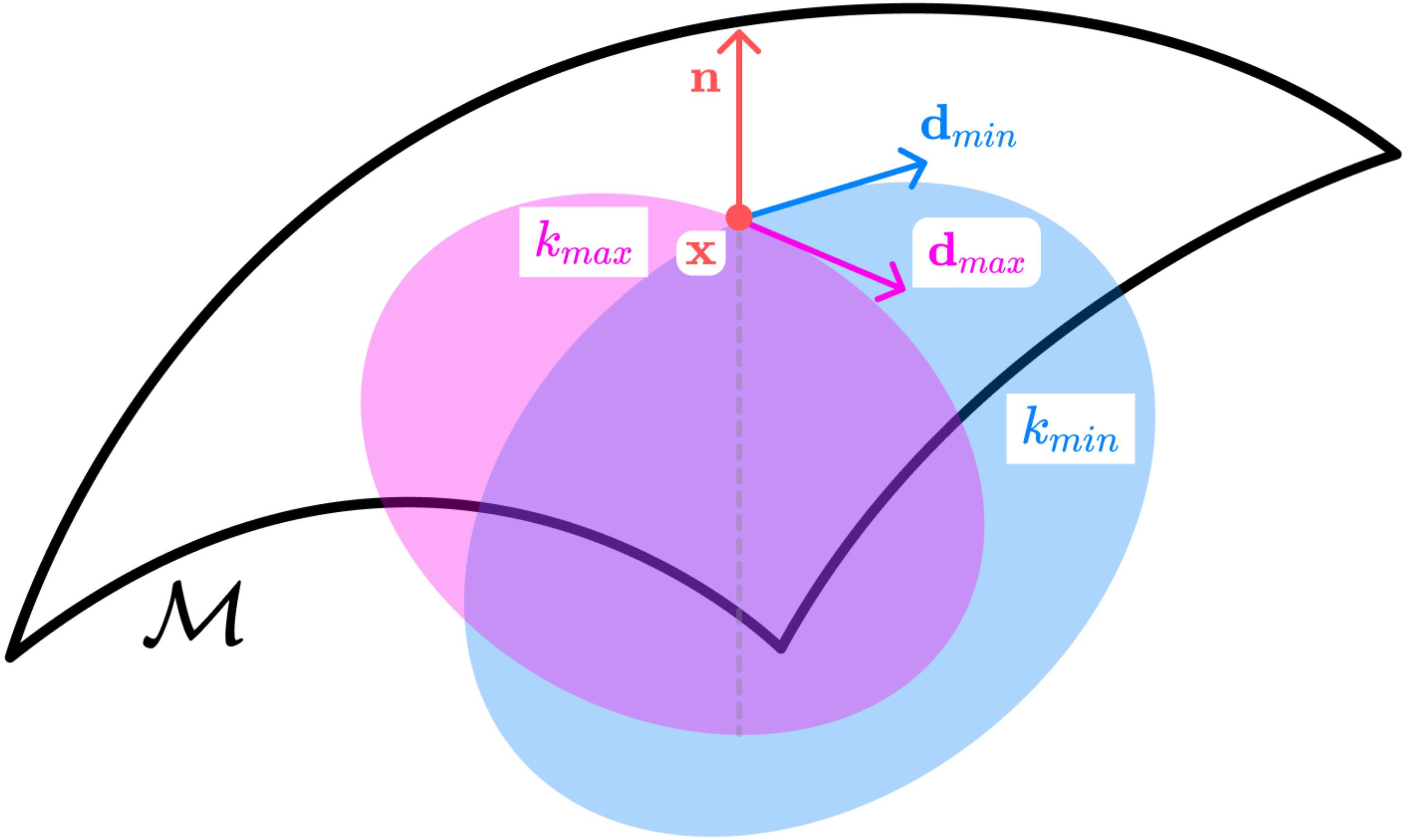
Weingarten Map

- **Decomposition** of the Weingarten map

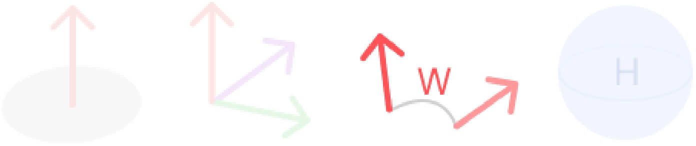
$$\mathrm{W} = \mathrm{U}\,\mathrm{D}\,\mathrm{U}^{\mathrm{T}}$$

$$\mathrm{U} = (\mathbf{d}_{min} \quad \mathbf{d}_{max})$$

$$\mathrm{D} = \begin{pmatrix} \mathrm{k}_{min} & 0 \\ 0 & \mathrm{k}_{max} \end{pmatrix}$$

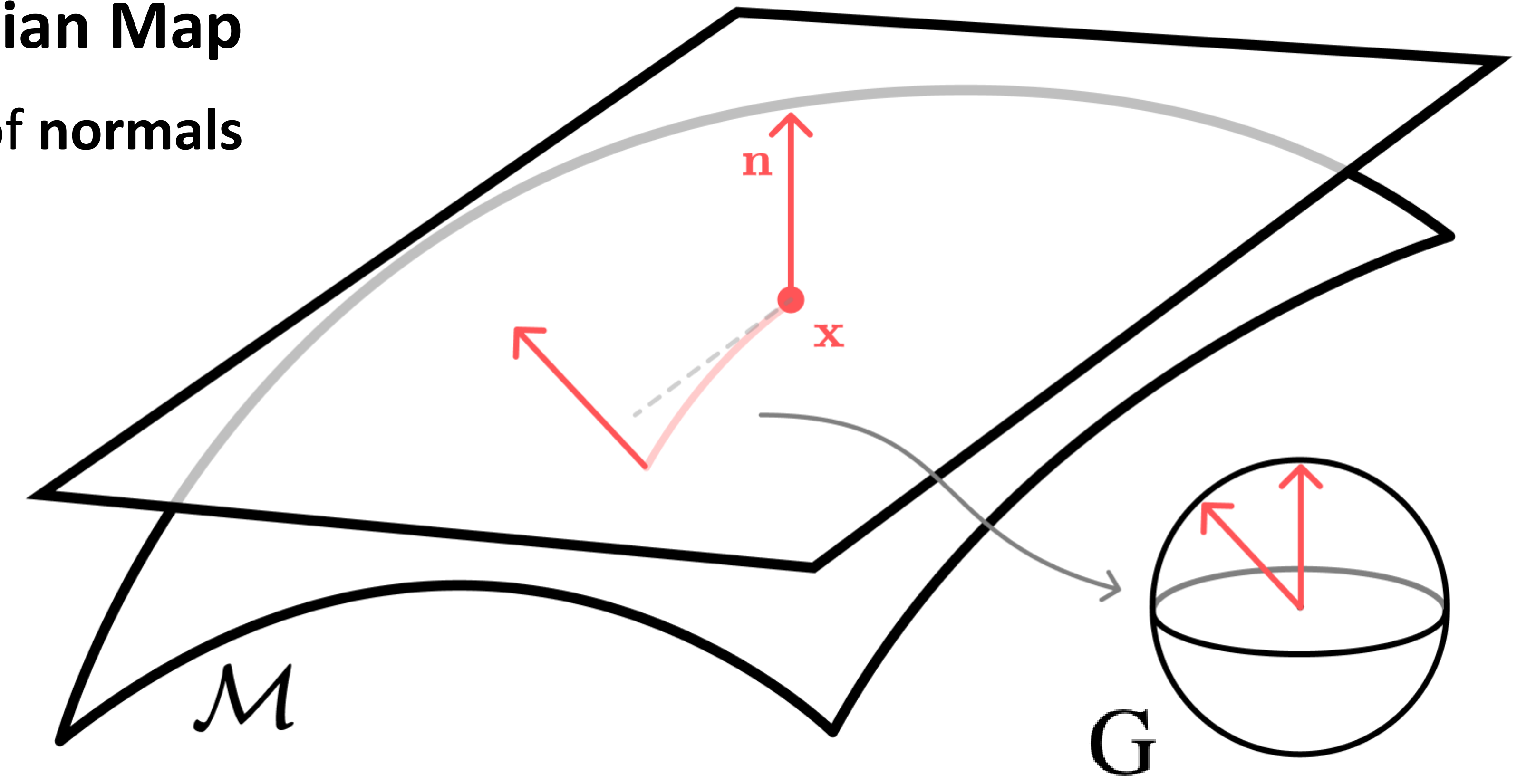


Weingarten Map – Other intuition



- Derivatives of the **Gaussian Map**
  - Observe the **variations** of **normals**

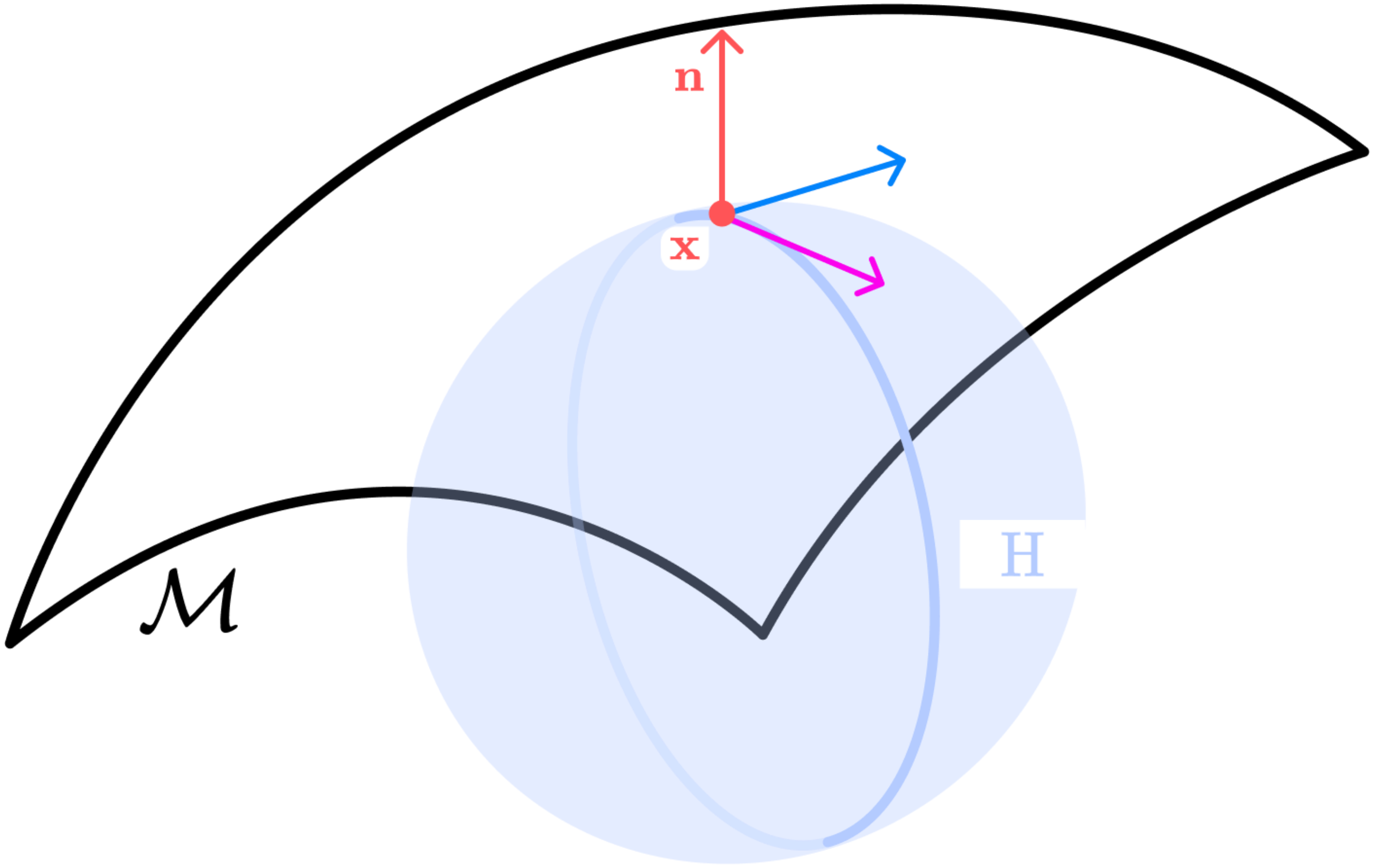
$$W = -D_{\mathbf{x}}G$$



Mean Curvature

- **Averaging** the principal curvatures

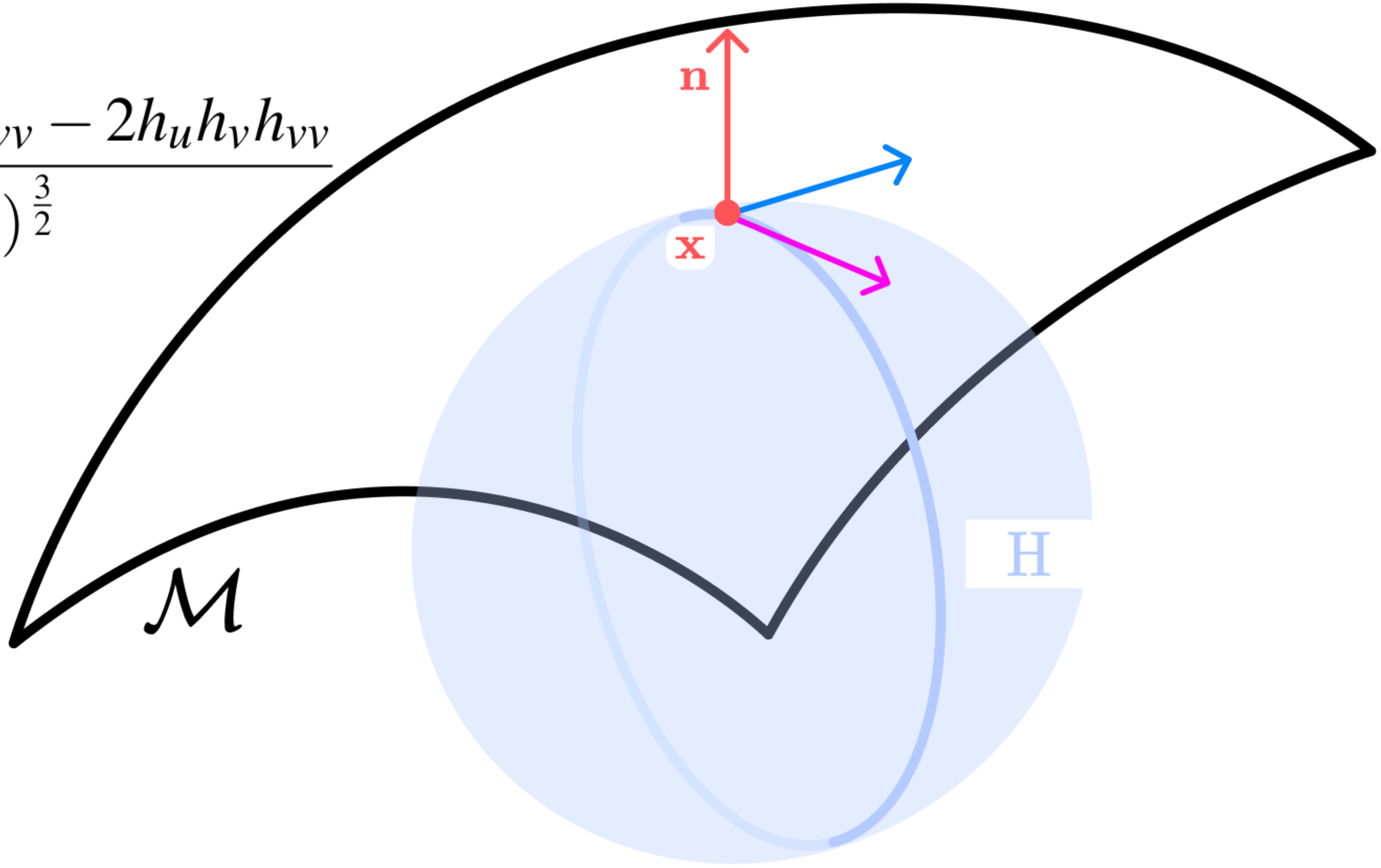
$$H = \frac{1}{2}\,\mathrm{trace}(W) = \frac{1}{2}\,\mathrm{trace}(D)$$



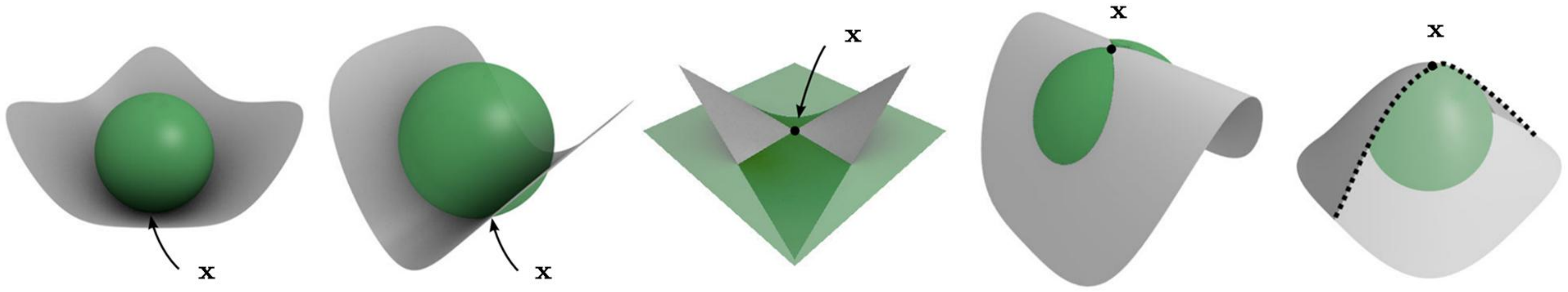
Mean Curvature

- Directly with the partial derivatives of h

$$H = \frac{(1+h_u^2)h_{uu} + (1+h_v^2)h_{vv} - 2h_u h_v h_{vv}}{2(h_u^2+h_v^2+1)^{\frac{3}{2}}}$$



Fundamentals – Examples



$$k_{min} = \alpha,\quad k_{max} = \alpha,\quad H = \alpha$$

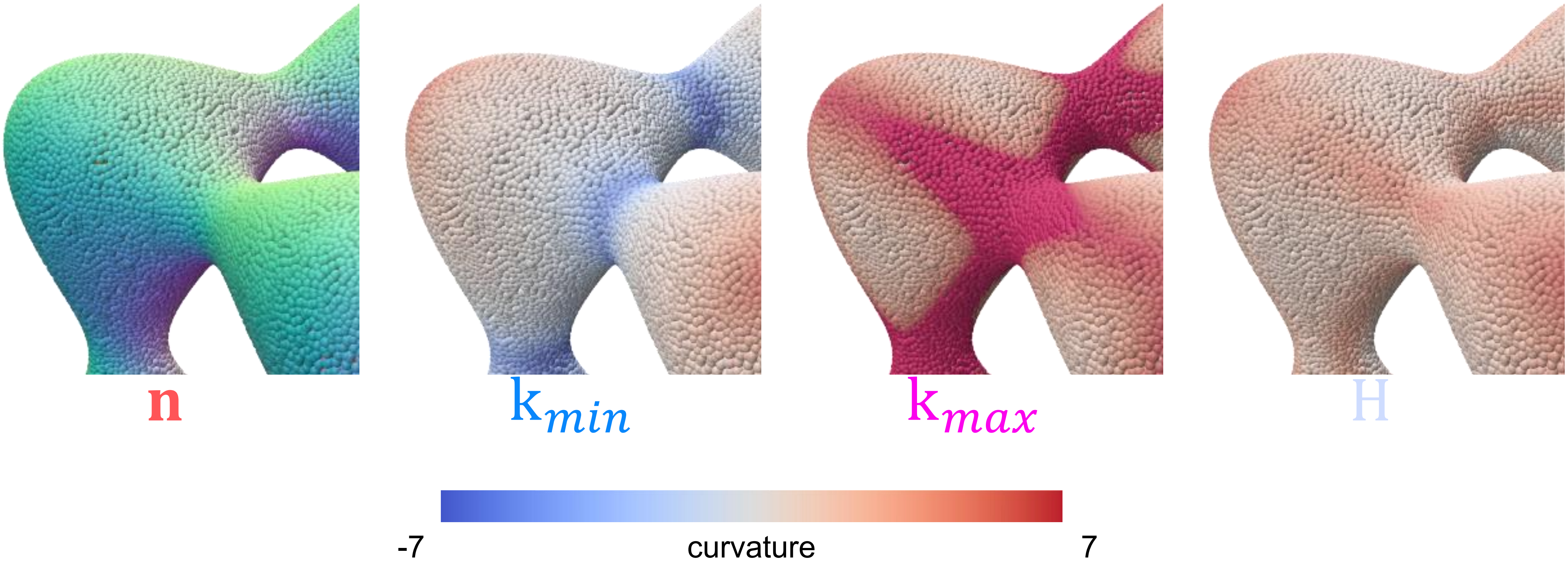
$$k_{min} = \alpha,\quad k_{max} = 0,\quad H = \frac{1}{2}\alpha$$

$$k_{min} = \alpha,\quad k_{max} = -\alpha,\quad H = 0$$

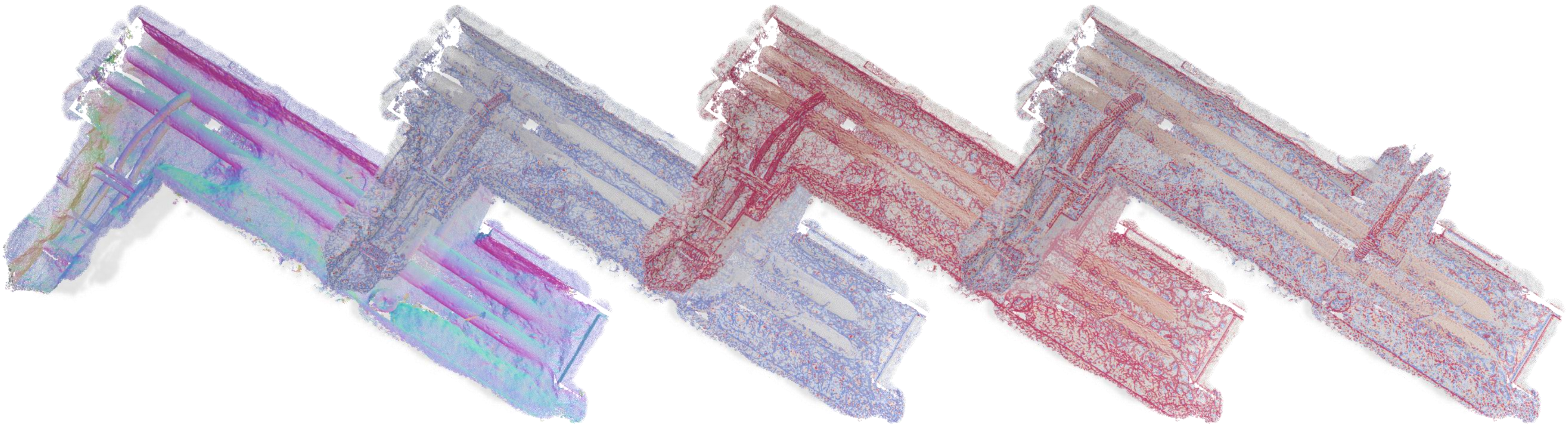
$$k_{min} = 0,\quad k_{max} = -\alpha,\quad H = -\frac{1}{2}\alpha$$

$$k_{min} = -\alpha,\quad k_{max} = -\alpha,\quad H = -\alpha$$

Fundamentals – Examples



Fundamentals – Examples



$\mathbf{n}$ k_{min} k_{max} H

QUESTION TIME

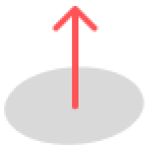
Context

Survey on differential estimators for 3d point clouds

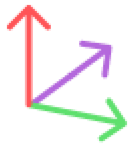
Methods

03

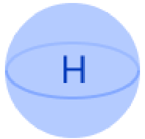
Methods - Notations



Normal estimation



Tangent frame estimation

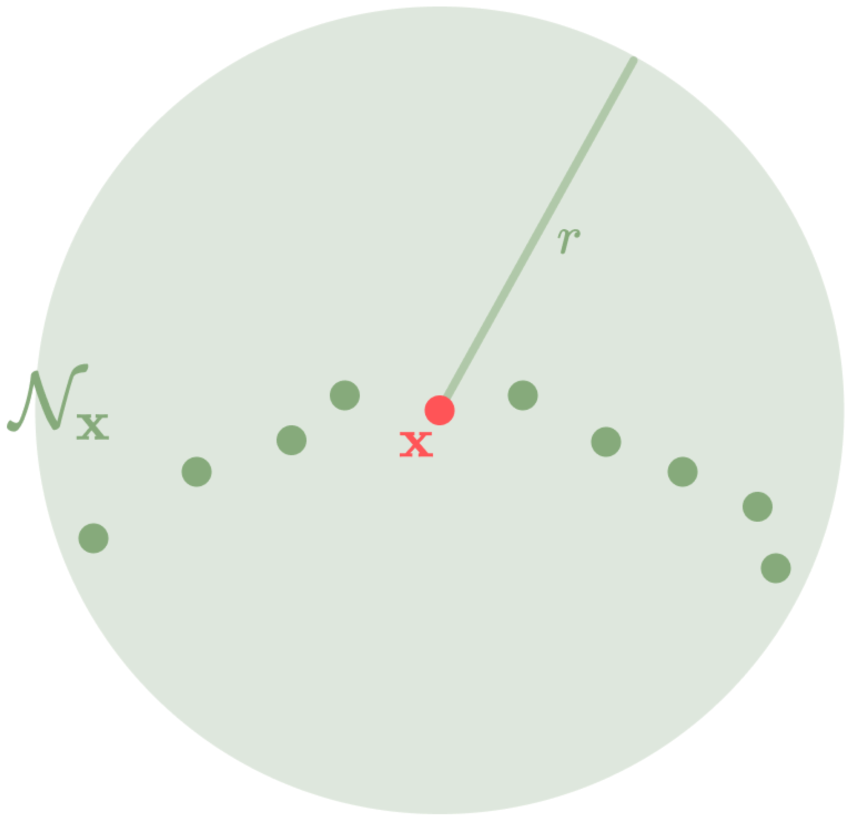


Mean curvature estimation

Weingarten map estimation
(principal curvature values and directions)

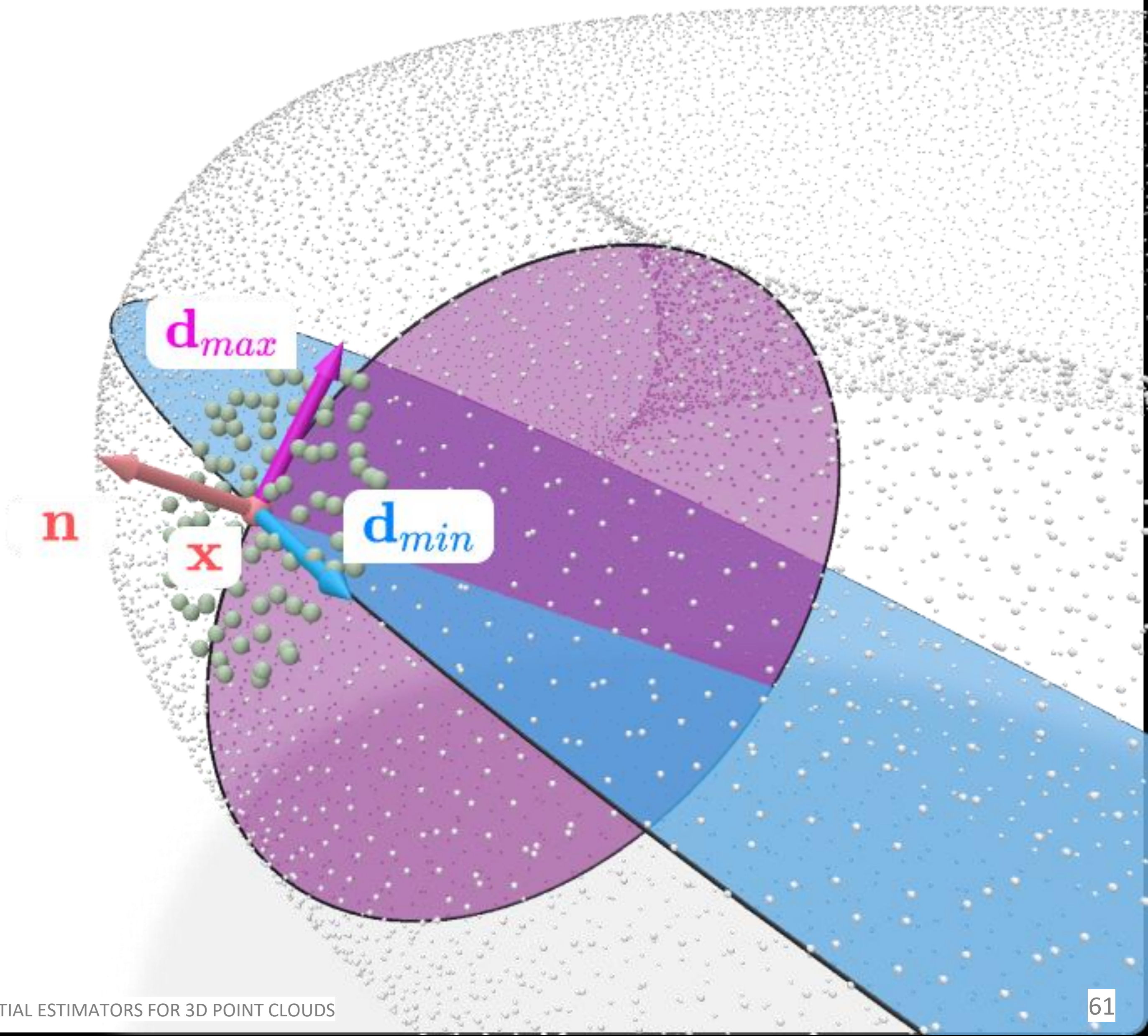
Methods – 2D example settings

- In the following sections

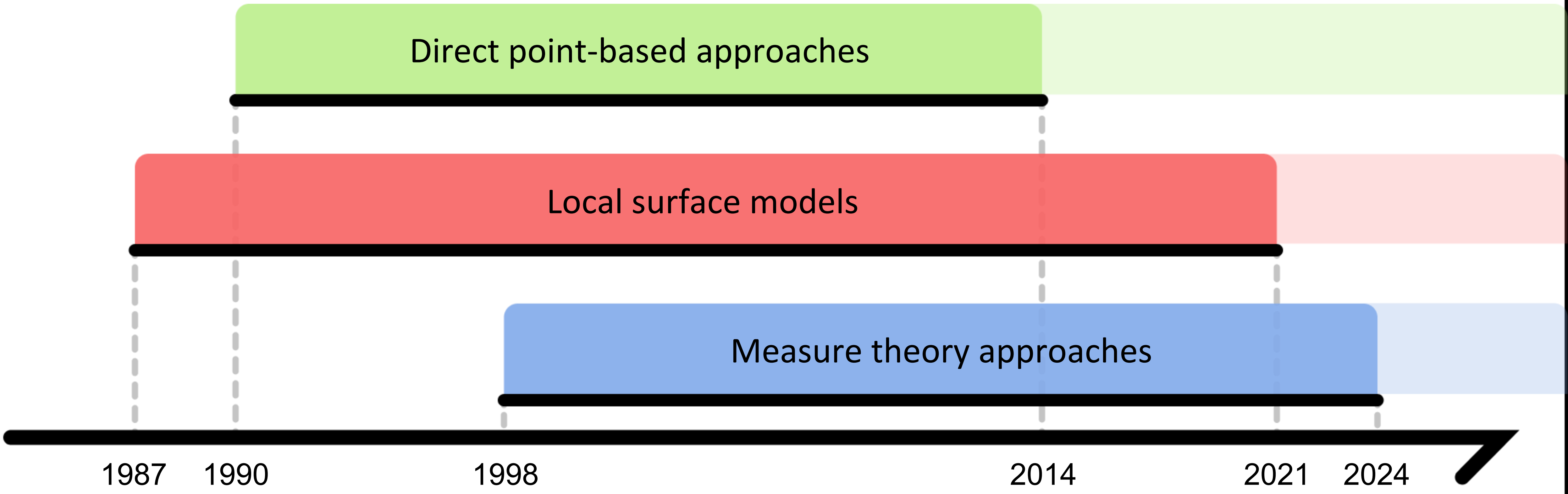


Methods – 3D example settings

- In the following sections

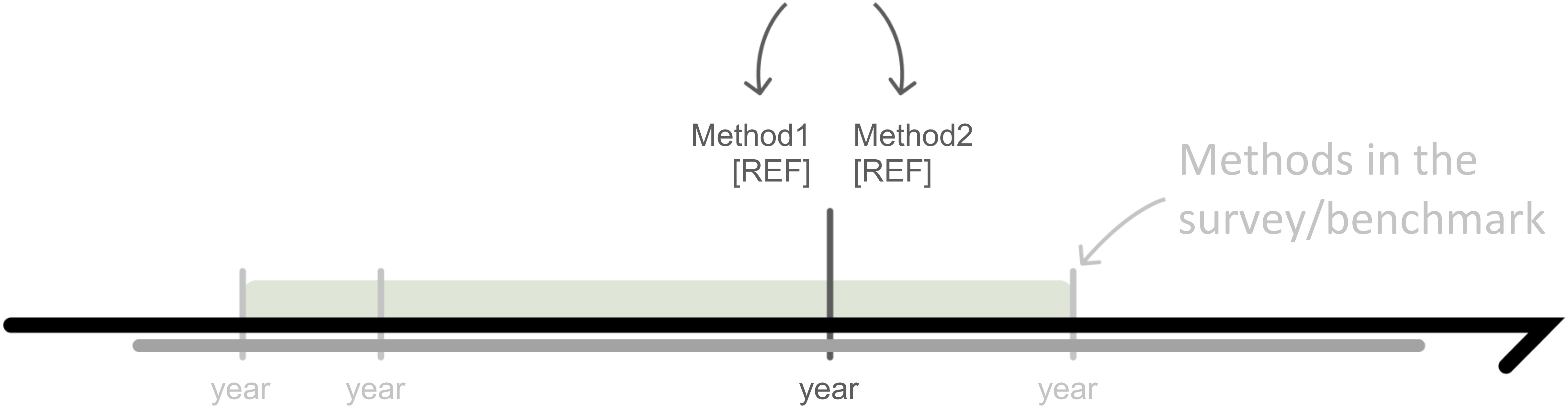


Methods - Overview



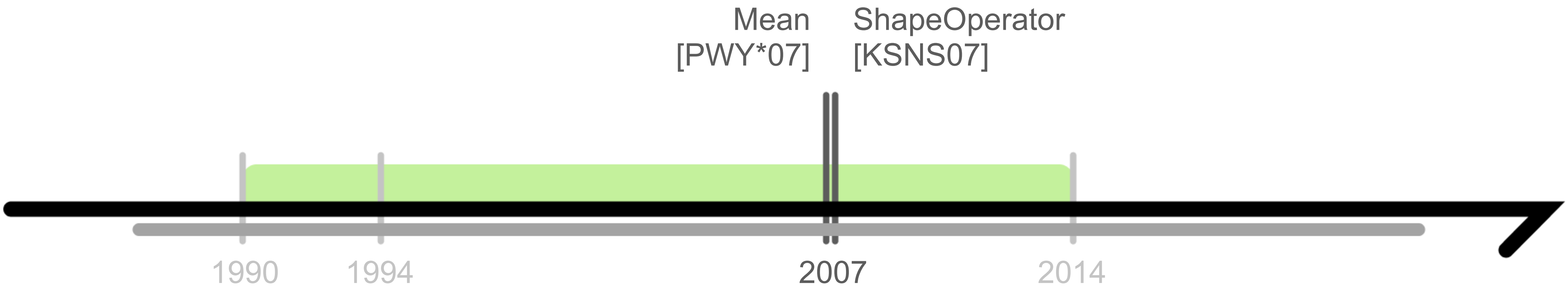
Methods – Display convention

Methods presented today



Methods – Direct Point-Based Approaches

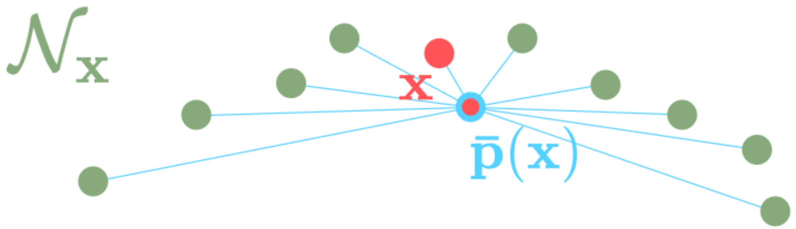
- Statistical approaches
  - Averages
  - Covariance matrices



Methods – Mean

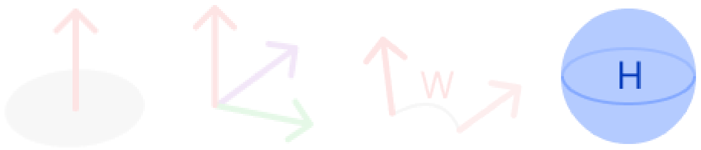
- The barycenter is linked to the mean curvature [PWY*07]

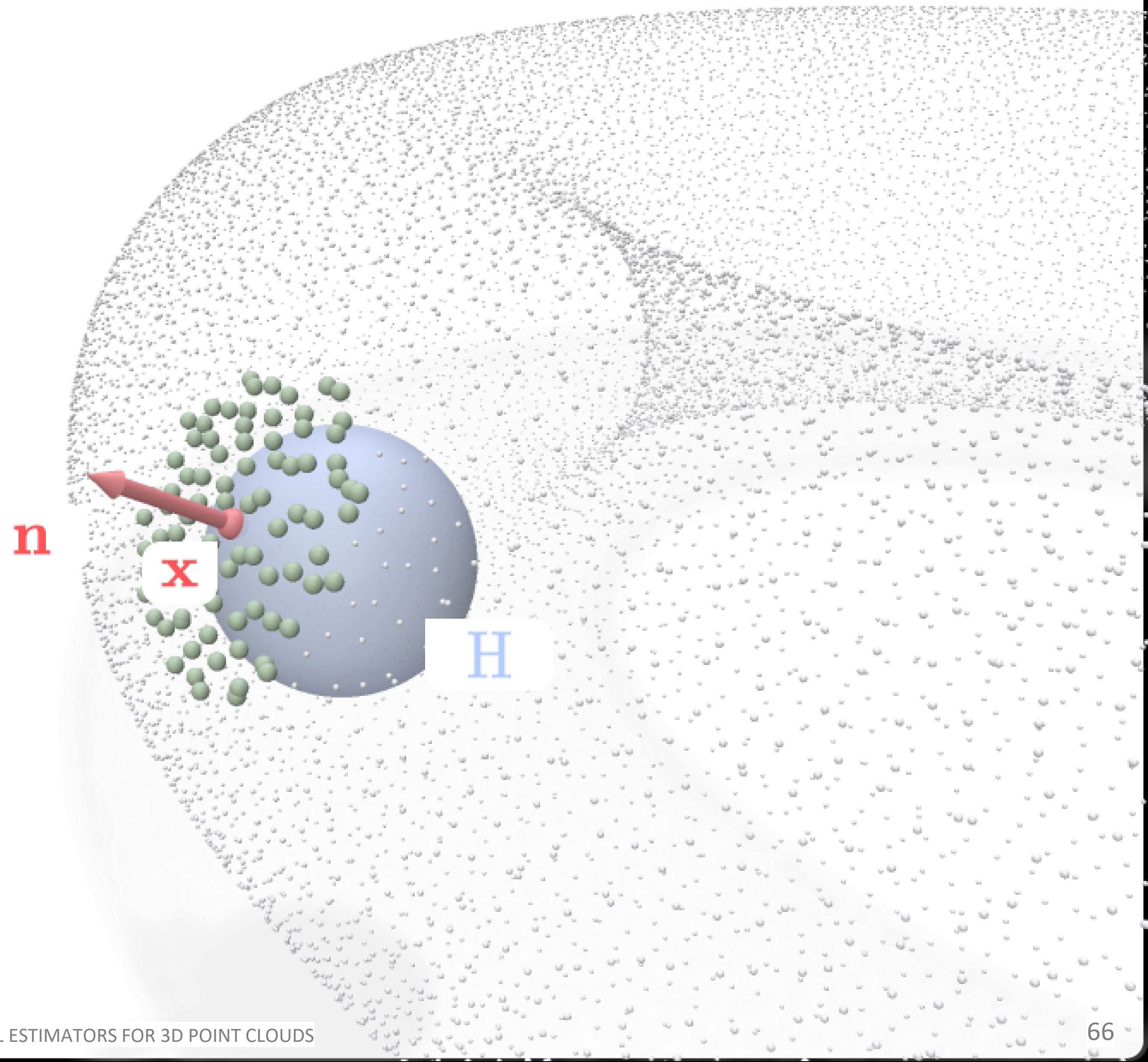
$$\bar{\mathrm{H}}(\mathbf{x}) = \frac{4\,\|\bar{\mathbf{p}}(\mathbf{x}) - \mathbf{x}\|}{r^2}$$



Methods – Mean

- Estimate
  - Mean curvature





Methods – ShapeOperator [KSNS07]

- Least-squares regression of the Weingarten Map
- Maps 2D directions in the tangent plane to 2D normal variations

$$W = F_{I}^{-1} F_{II}$$

Methods – ShapeOperator [KSNS07]

- Least-squares regression of the Weingarten Map
- Maps 2D directions in the tangent plane to 2D normal variations

$$W = F_I^{-1} F_{II}$$

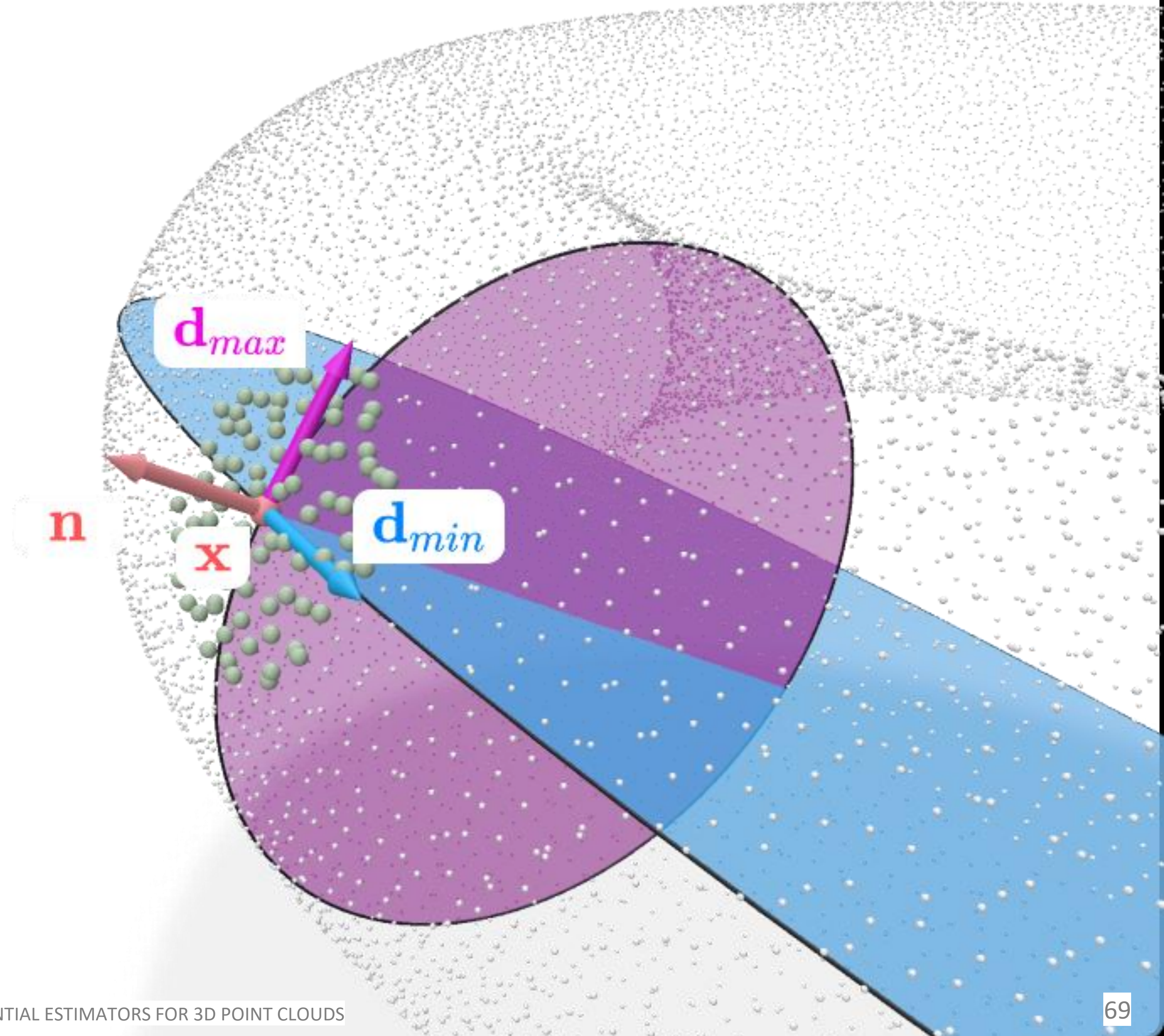
$$\bar{W}(\mathbf{x}) = \left(\sum_{i \in \mathcal{N}_\mathbf{x}} w_i \mathbf{d}_i \mathbf{d}_i^T \right)^{-1} \left(\sum_{i \in \mathcal{N}_\mathbf{x}} w_i \mathbf{v}_i \mathbf{v}_i^T \right)$$

$$\mathbf{d}_i = \bar{P}(\mathbf{x})^T (\mathbf{p}_i - \mathbf{x})$$ (position variation)

$$\mathbf{v}_i = \bar{P}(\mathbf{x})^T (\mathbf{n}_i - \mathbf{n})$$ (normal variation)

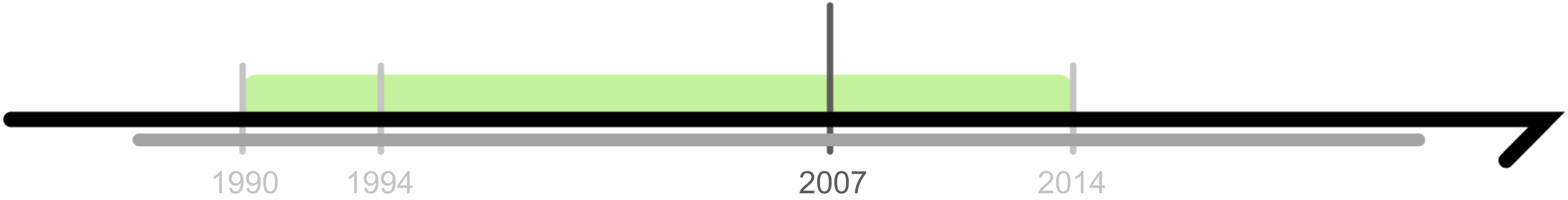
Methods – ShapeOperator [KSNS07]

- Estimate
  - Weingarten Map
- Requirements
  - Oriented normal vectors



Methods – Direct Point-Based Approaches

- Capture the geometry directly without reconstructing the explicit surface



Methods – Direct Point-Based Approaches

Pros

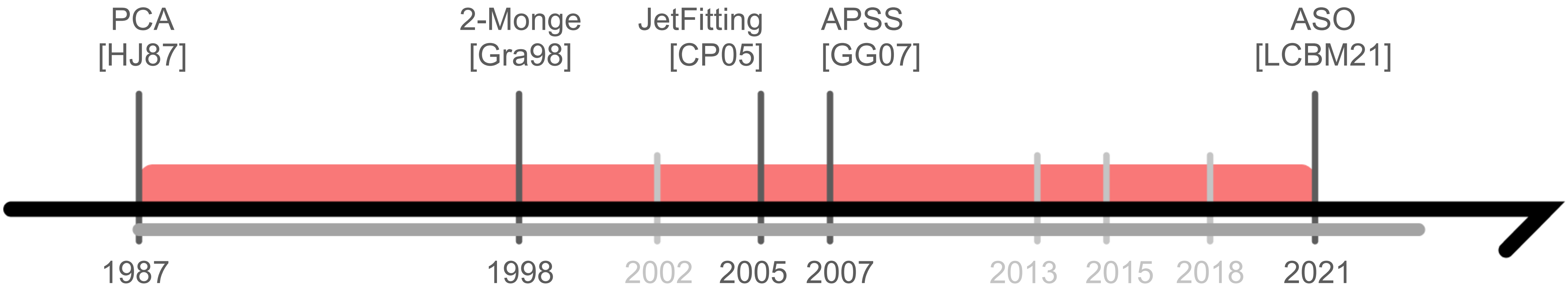
- Simple formulas, efficient
- Good to estimate principal directions

Cons

- Noise sensitive
- Lack of precision for curvature estimation (except for the ShapeOperator)

Methods – Local surface models

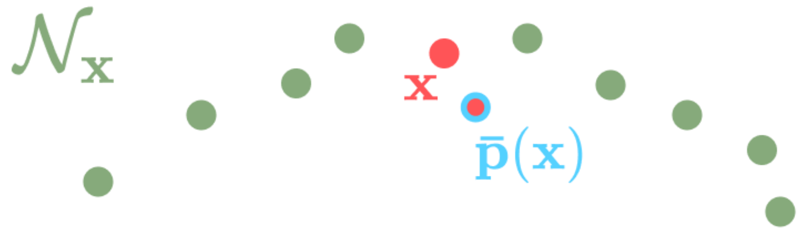
- 2.5D height field fitting
- 3D surface fitting



Methods – PCA [HJ87]

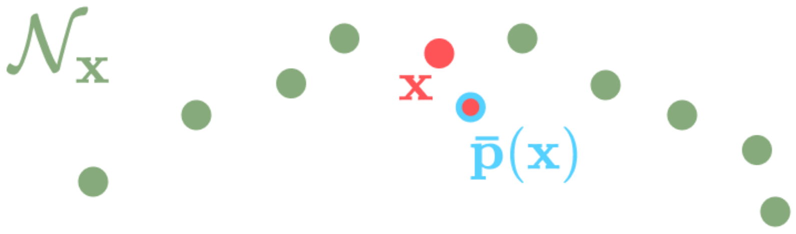
- Find orthogonal directions of position's variance

$$C(\mathbf{x}) = \frac{1}{\sum_{i\in\mathcal{N}_\mathbf{x}} w_i} \sum_{i\in\mathcal{N}_\mathbf{x}} w_i \left(\mathbf{p}_i - \bar{\mathbf{p}}(\mathbf{x})\right)\left(\mathbf{p}_i - \bar{\mathbf{p}}(\mathbf{x})\right)^T$$



Methods – PCA [HJ87]

- Find orthogonal directions of position’s variance
- Tangent frame / normal estimation



$$\mathrm{C}(\mathbf{x}) = \frac{1}{\sum_{i \in \mathcal{N}_{\mathbf{x}}} w_i} \sum_{i \in \mathcal{N}_{\mathbf{x}}} w_i \left(\mathbf{p}_i - \bar{\mathbf{p}}(\mathbf{x})\right) \left(\mathbf{p}_i - \bar{\mathbf{p}}(\mathbf{x})\right)^T$$

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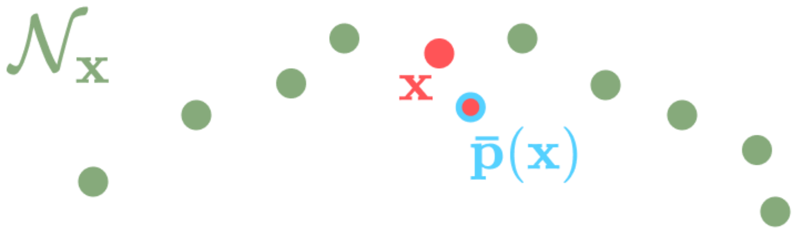

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$\lambda_0^{\mathrm{C}} \leq \lambda_1^{\mathrm{C}} \leq \lambda_2^{\mathrm{C}}$ eigenvalues (related to position's variance)

$\mathbf{u}_0^{\mathrm{C}}$ $\mathbf{u}_1^{\mathrm{C}}$ $\mathbf{u}_2^{\mathrm{C}}$ eigenvectors (normal and frame directions)

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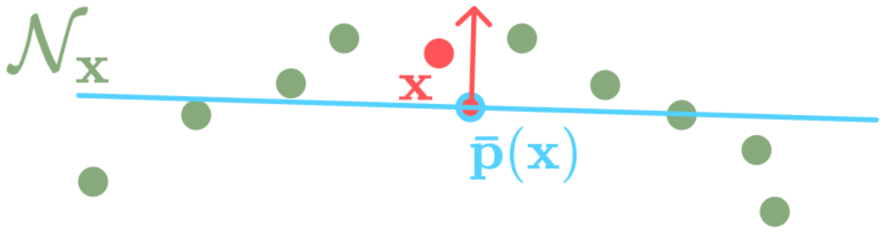

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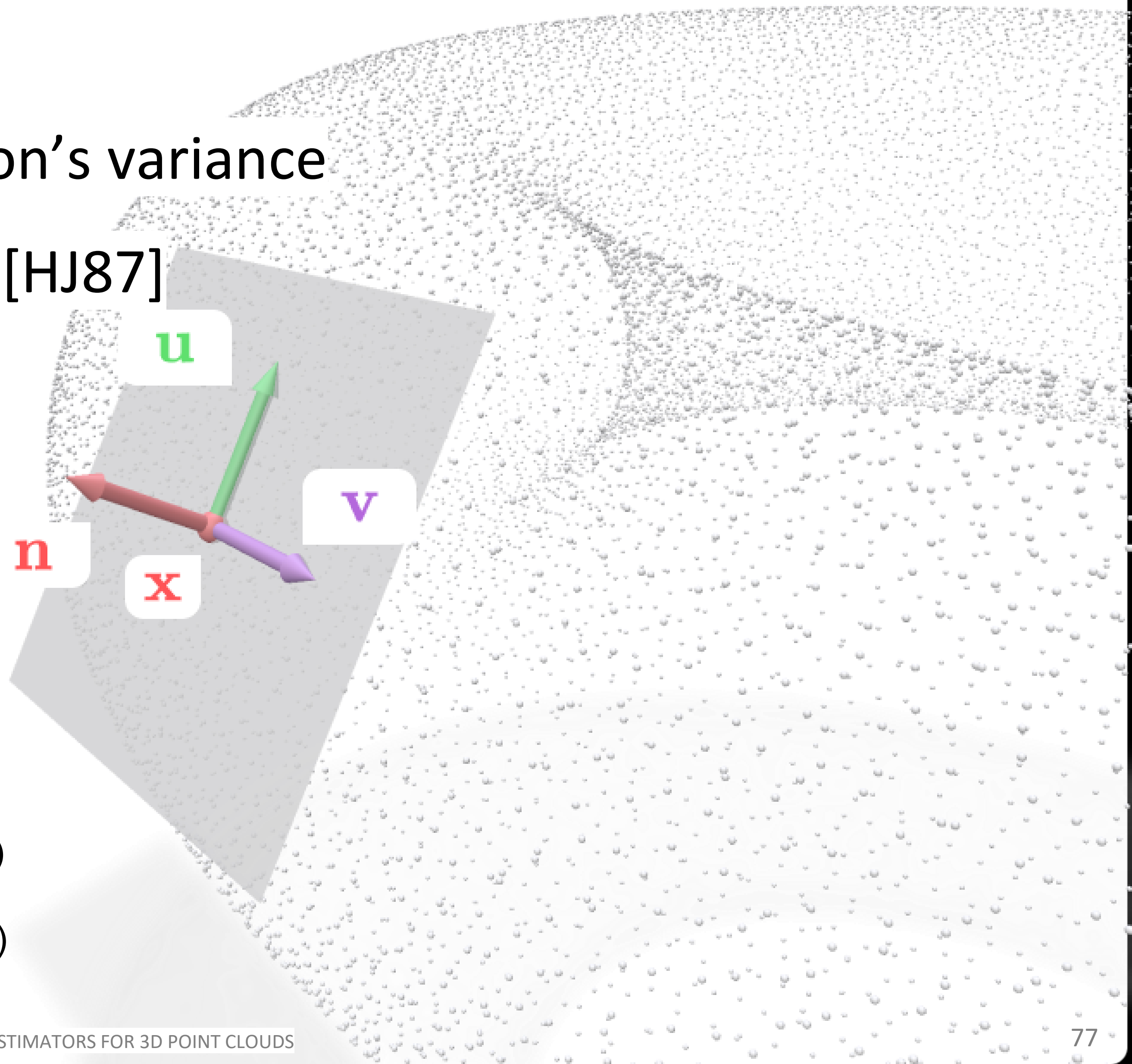
- Tangent frame / normal estimation [HJ87]



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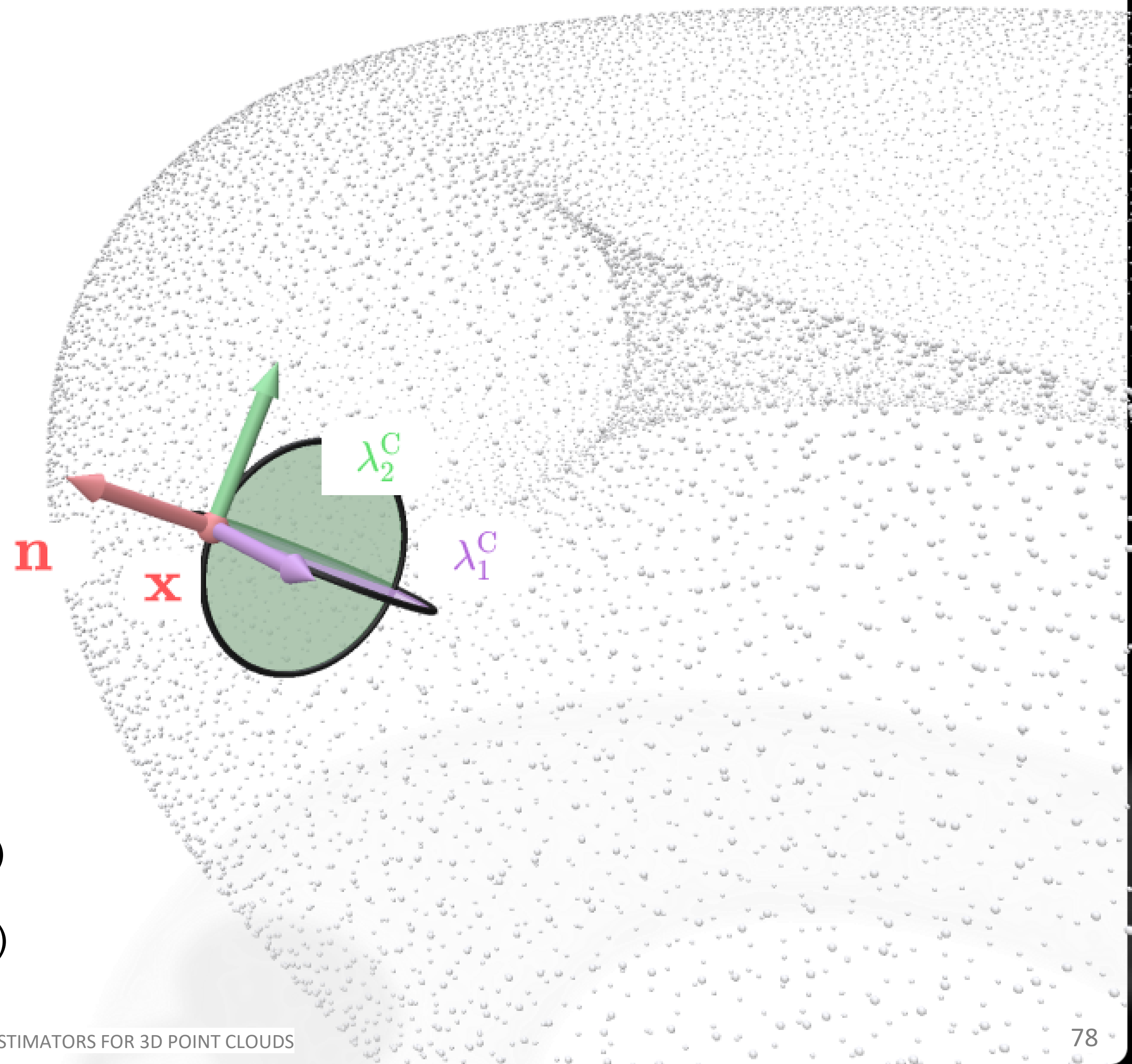
Methods – PCA [HJ87]

- Analysis of the eigenvalues
  - Not related to curvatures
  - Over-estimate the curvatures



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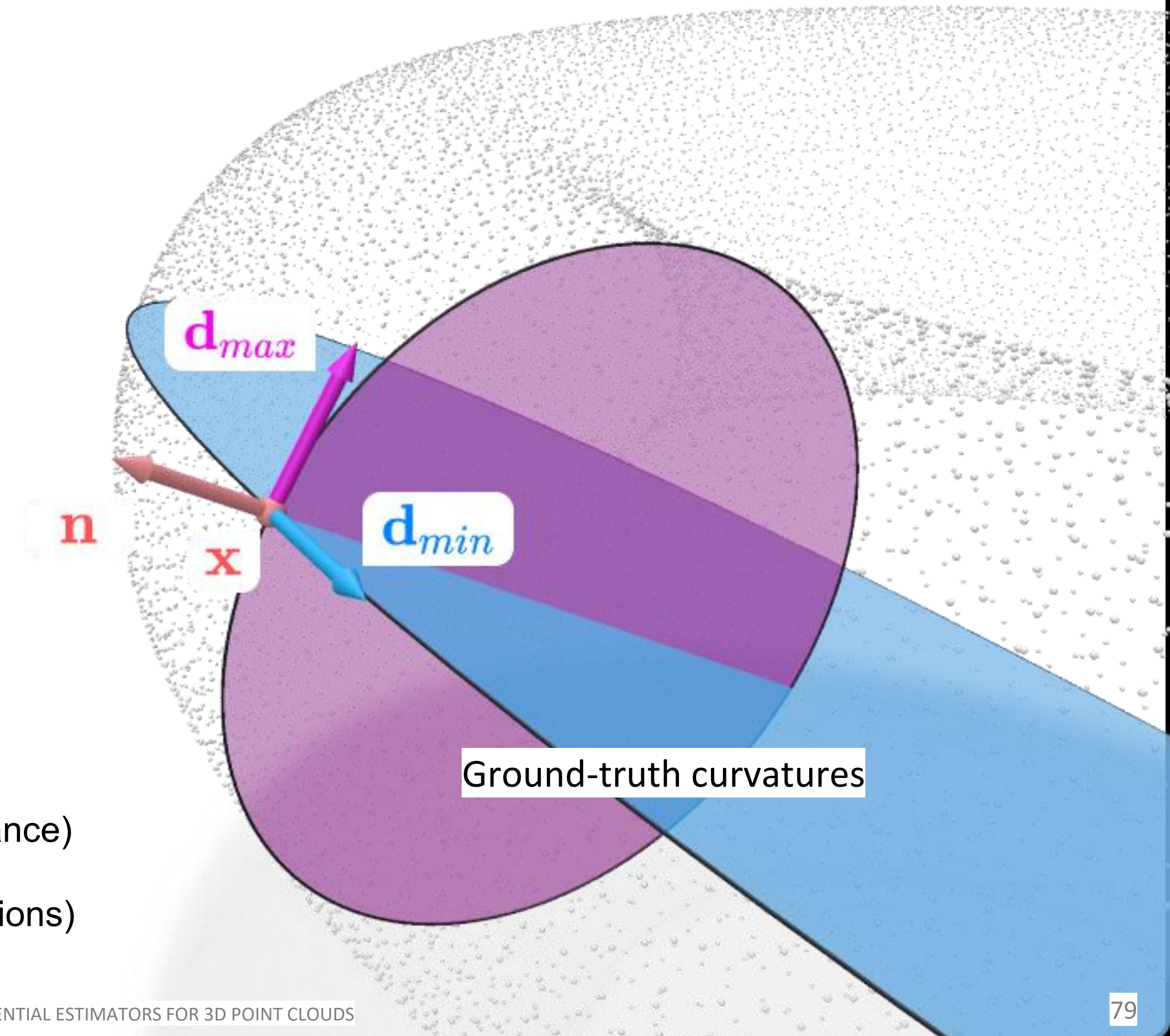

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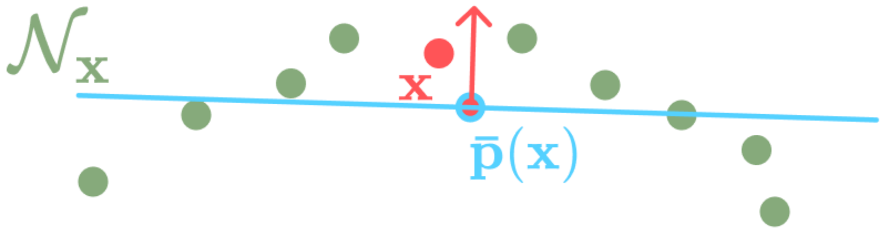
Methods – 2-Monge [Gra98]

- Monge patch fitting
  - 2nd order bivariate polynomial

$$h(\mathbf{q}) = u_c + \mathbf{u}_\ell^T \mathbf{q} + \mathbf{q}^T \mathrm{U}_q \mathbf{q}$$



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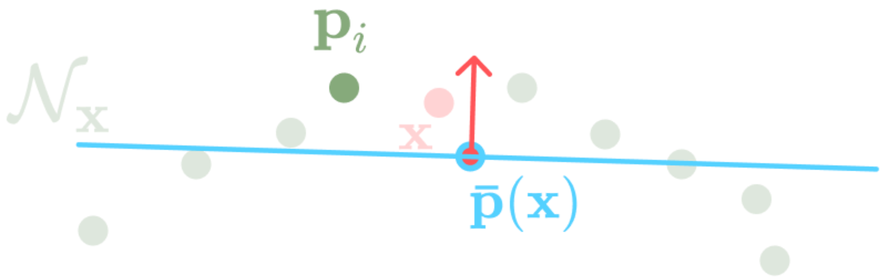
height to the plane
2D point coordinates

$$\begin{pmatrix} h_i \\ \mathbf{q}_i \end{pmatrix} = \bar{\mathrm{B}}(\mathbf{x})^T \left(\mathbf{p}_i - \bar{\mathbf{p}}(\mathbf{x})\right)$$



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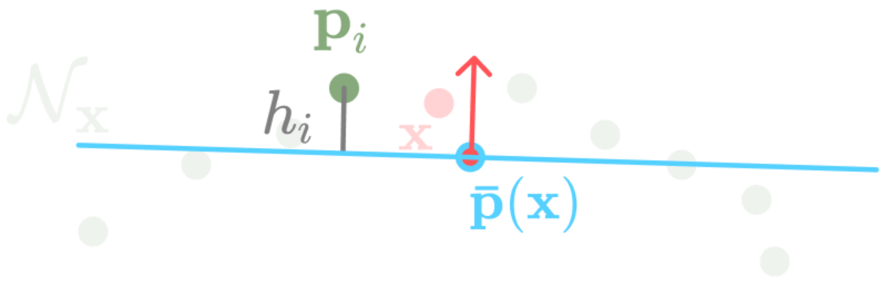

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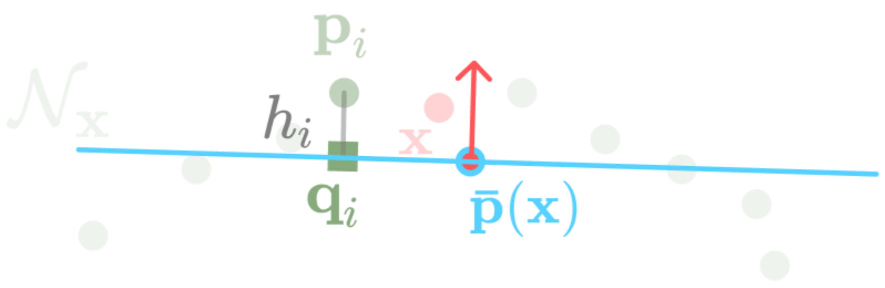

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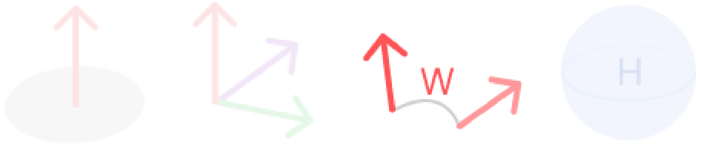

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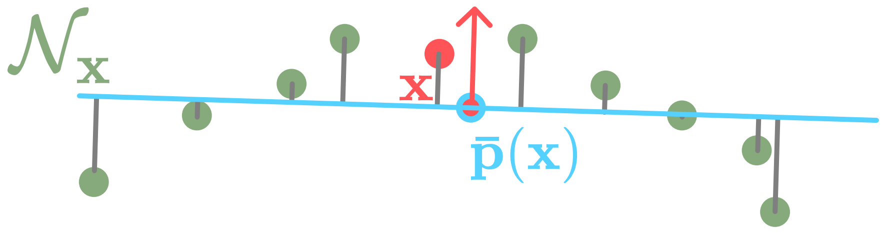

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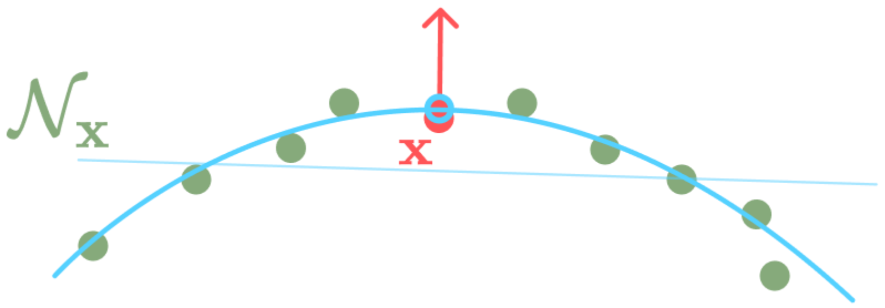
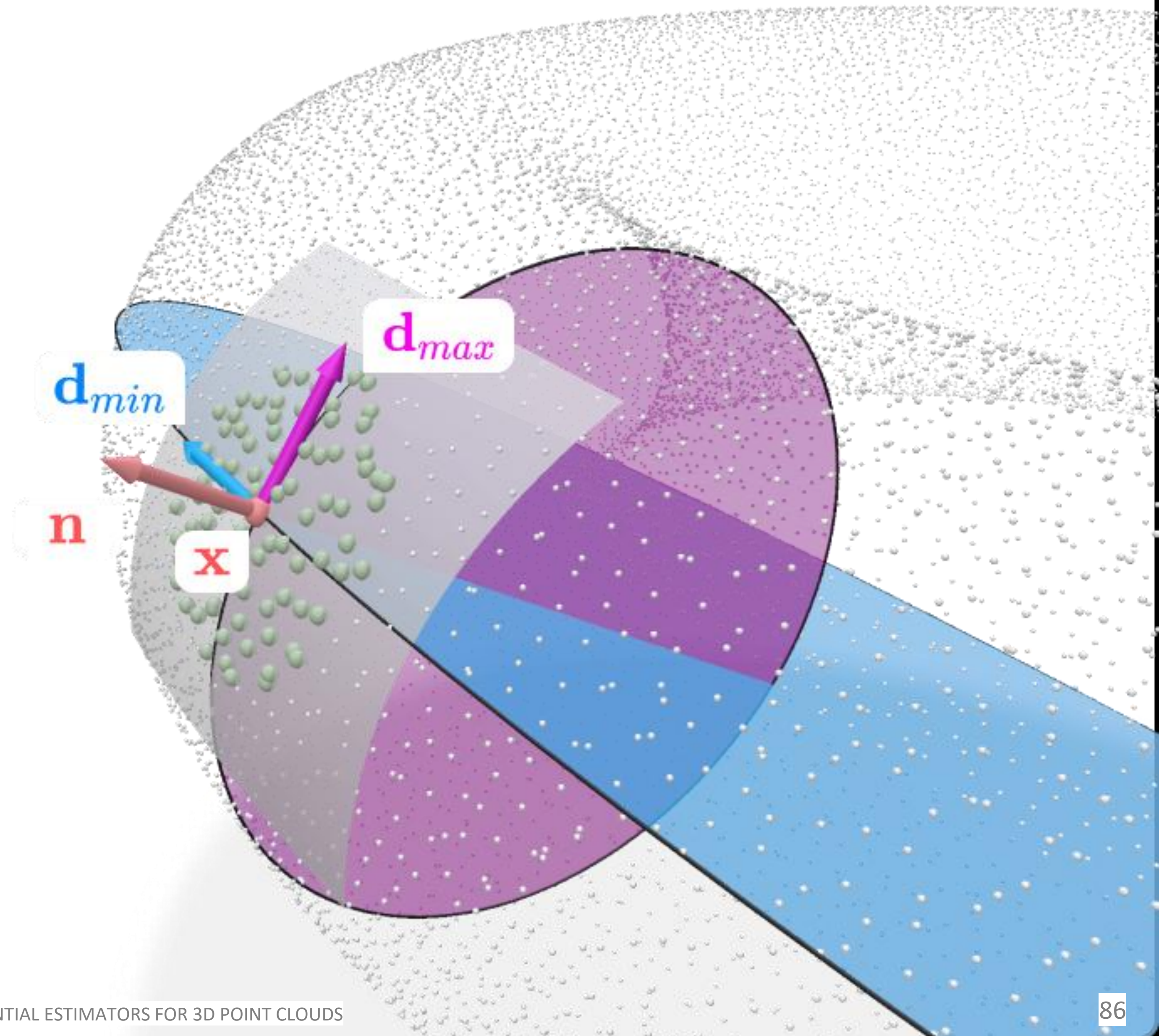


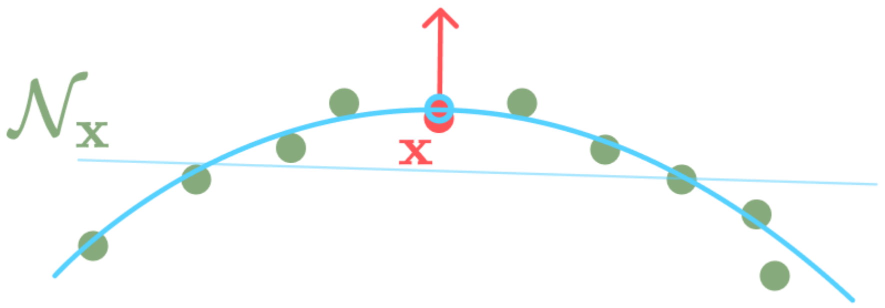
Methods – 2-Monge [Gra98]

- Estimate
  - Weingarten Map
- Requirements
  - Initial tangent frame



Methods – JetFitting [CP05]

- Monge patch fitting
  - Higher order bivariate polynomial
- Normal refinement



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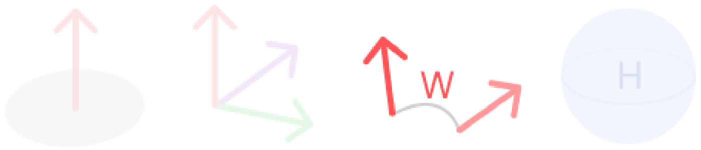

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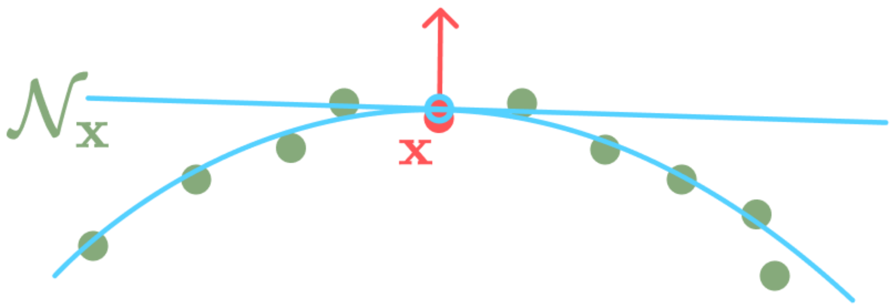

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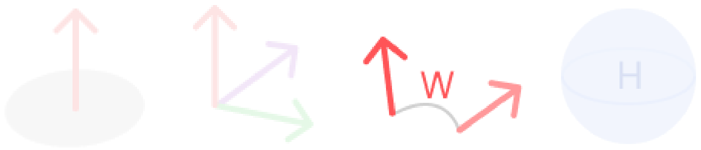

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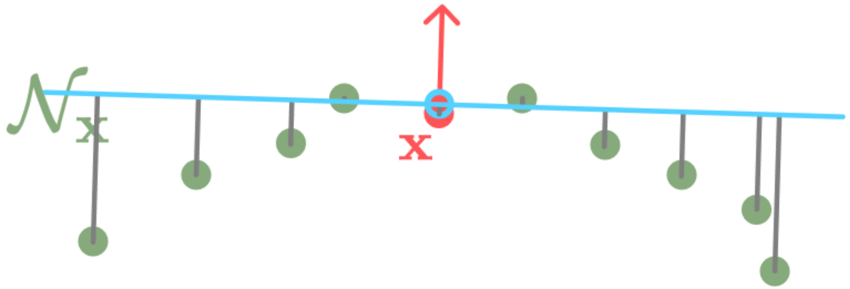

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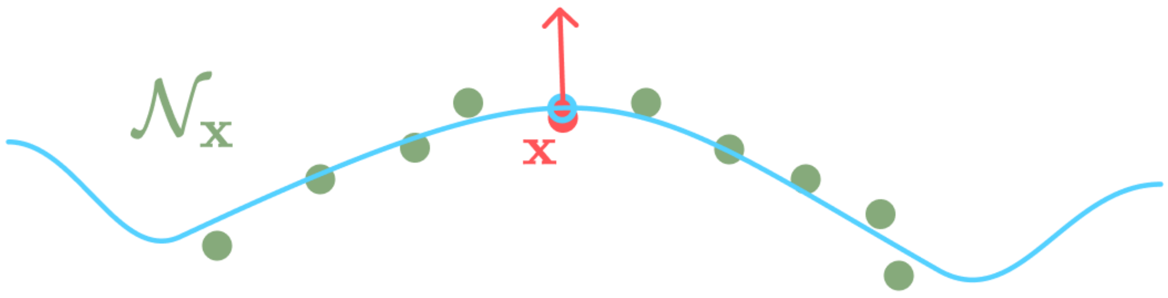

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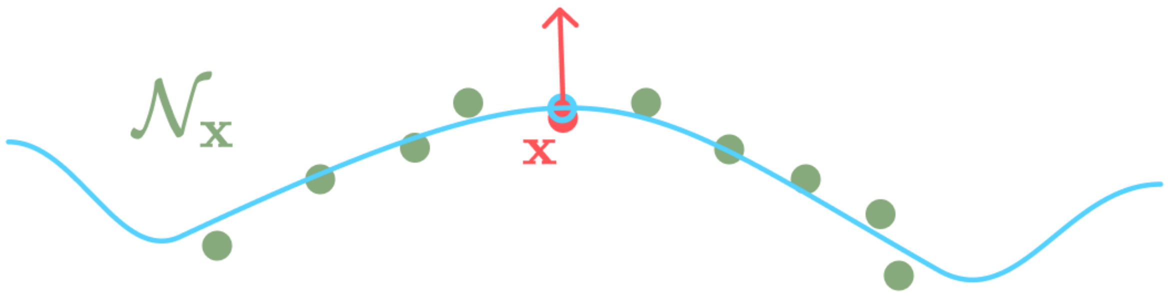

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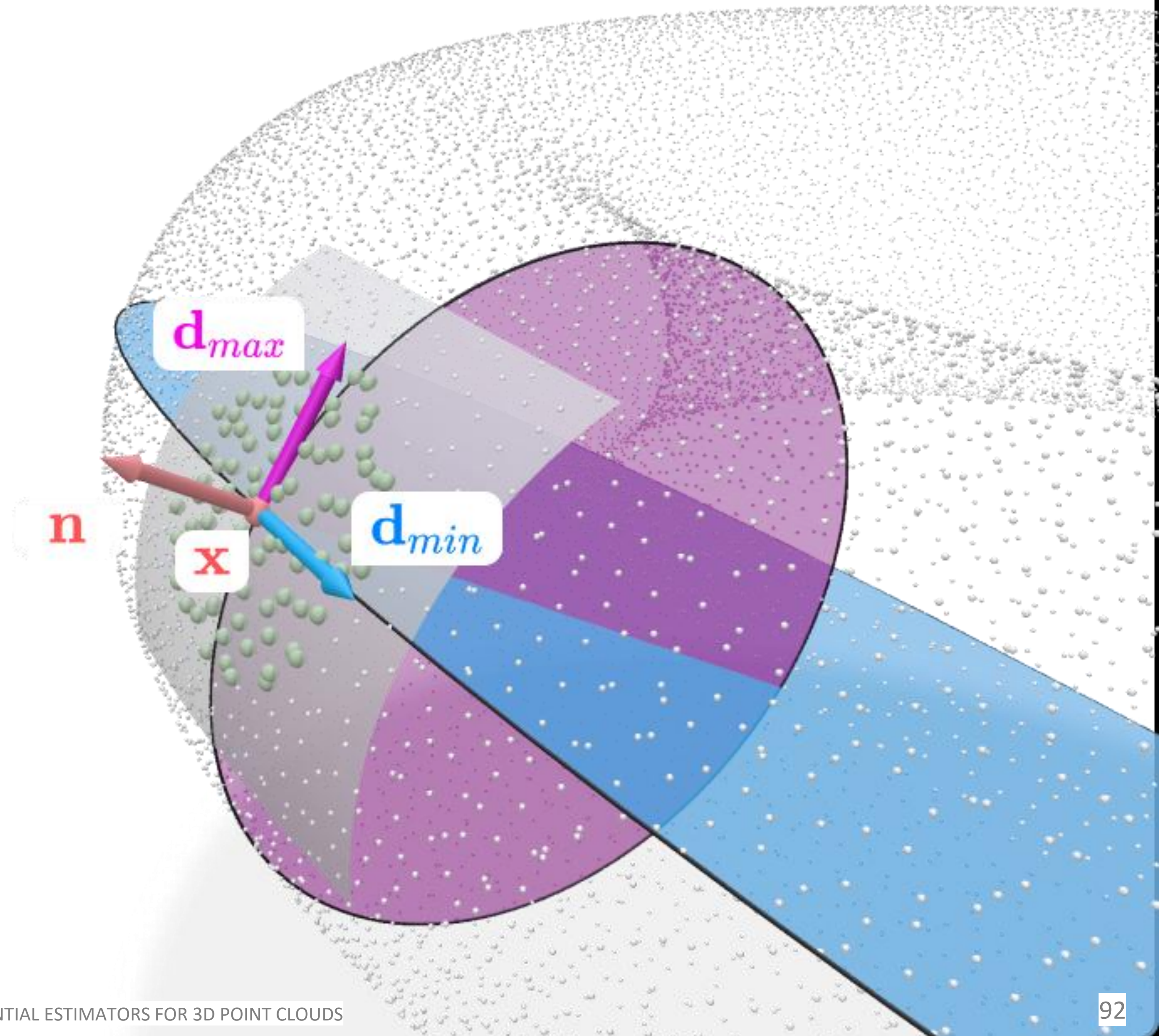
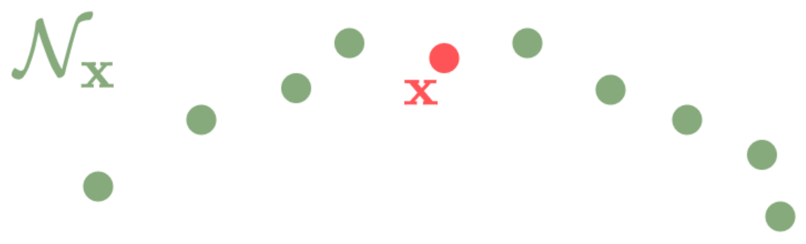


Methods – APSS [GG07]

- Algebraic sphere fitting
- Based on Pratt's work [Pra87]
- Closed-form formulation



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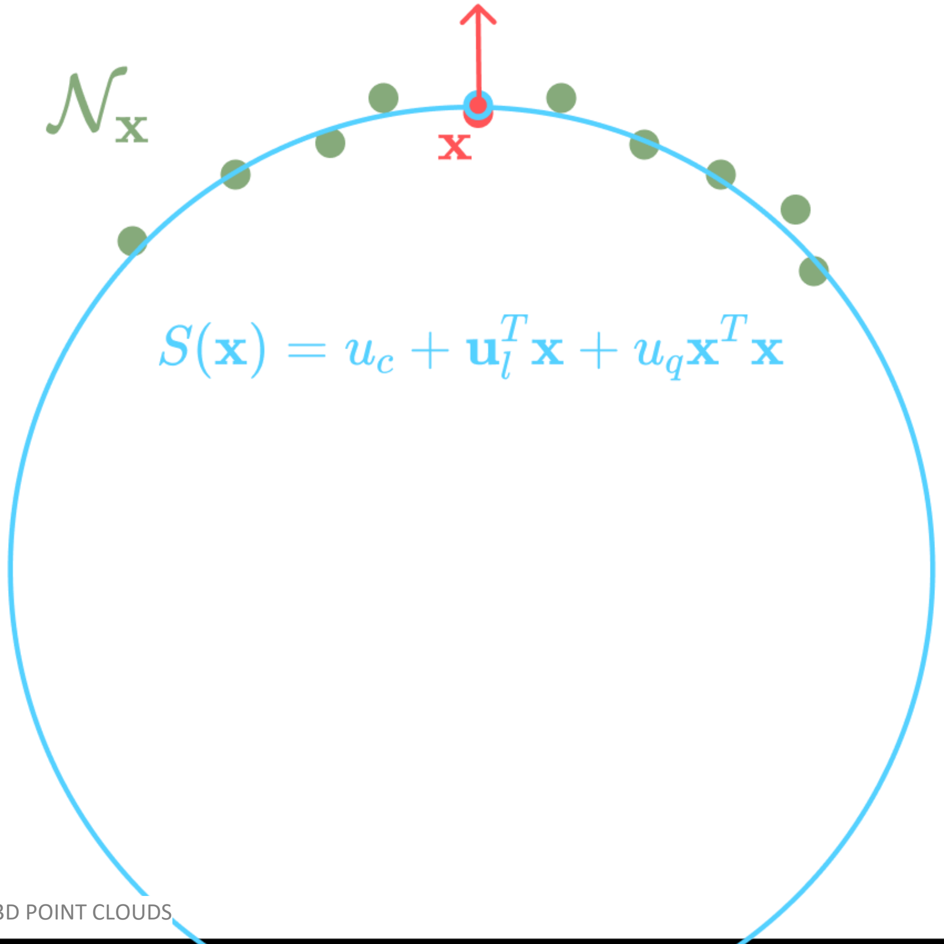

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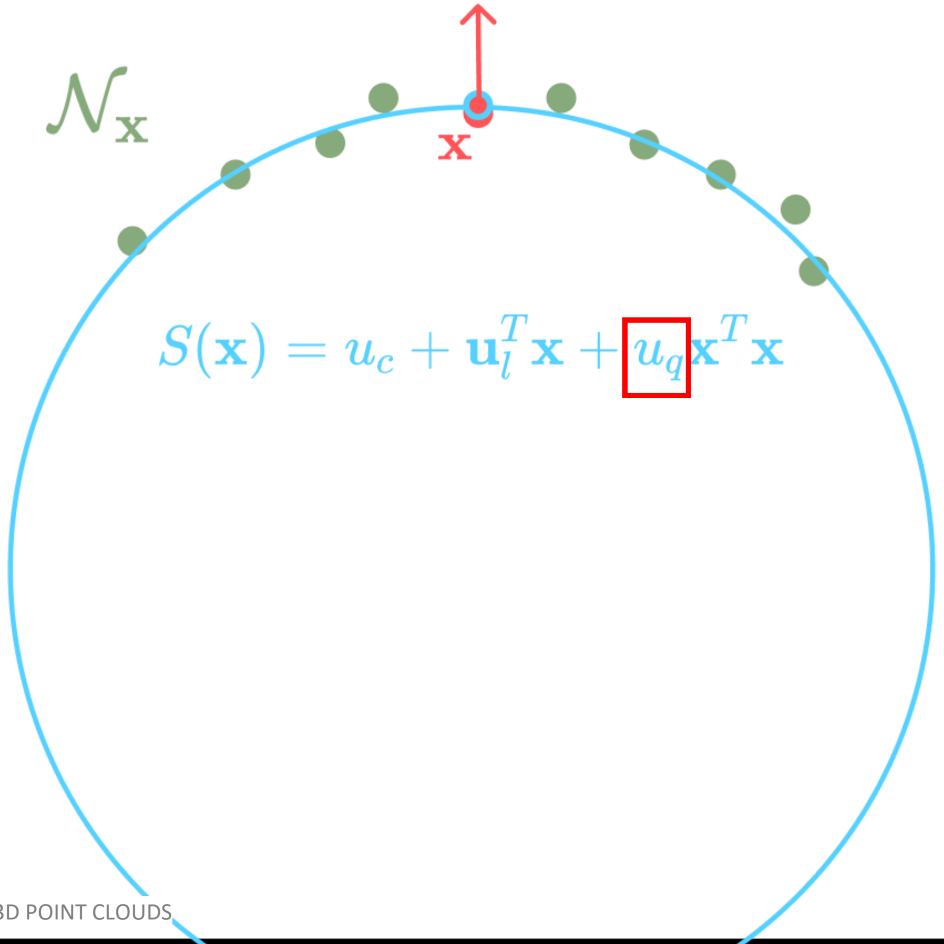

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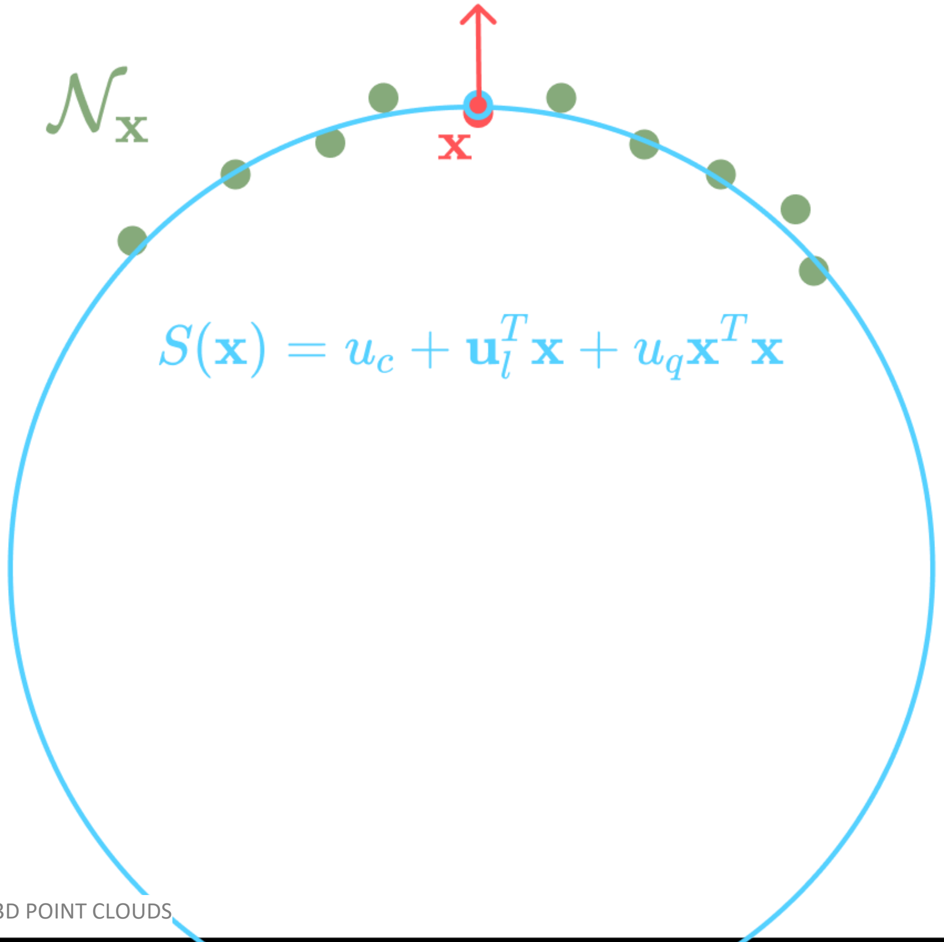

- Second-order derivative
  - $\nabla^2 S(\mathbf{x}) = 2\,u_q$

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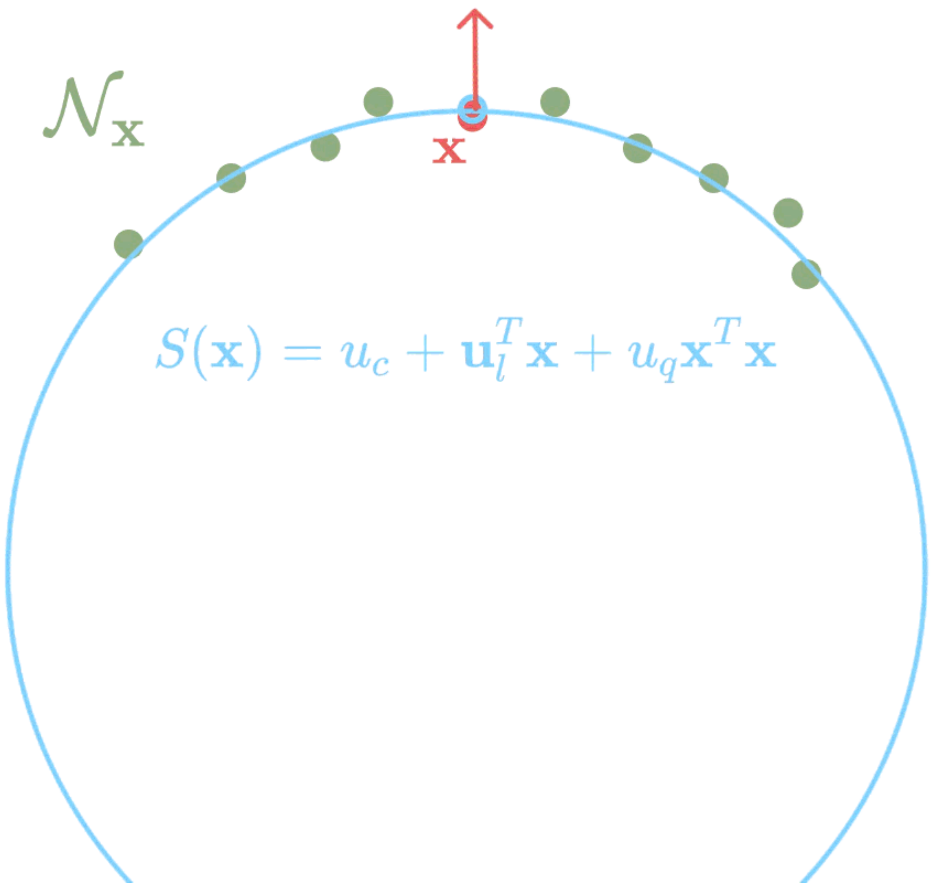
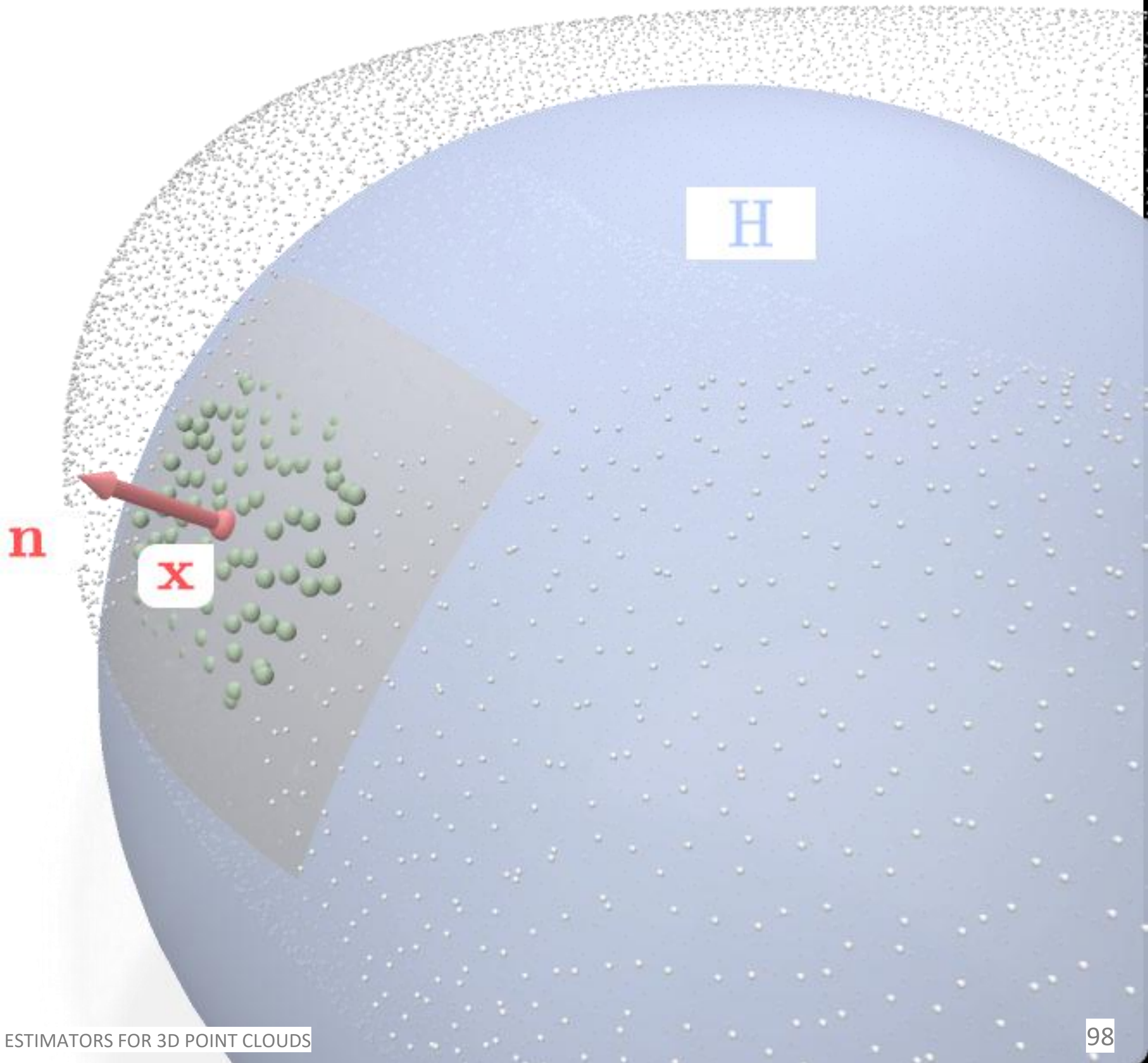
  - $\nabla^2 S(\mathbf{x}) = 2\, u_q$
  - Mean curvature



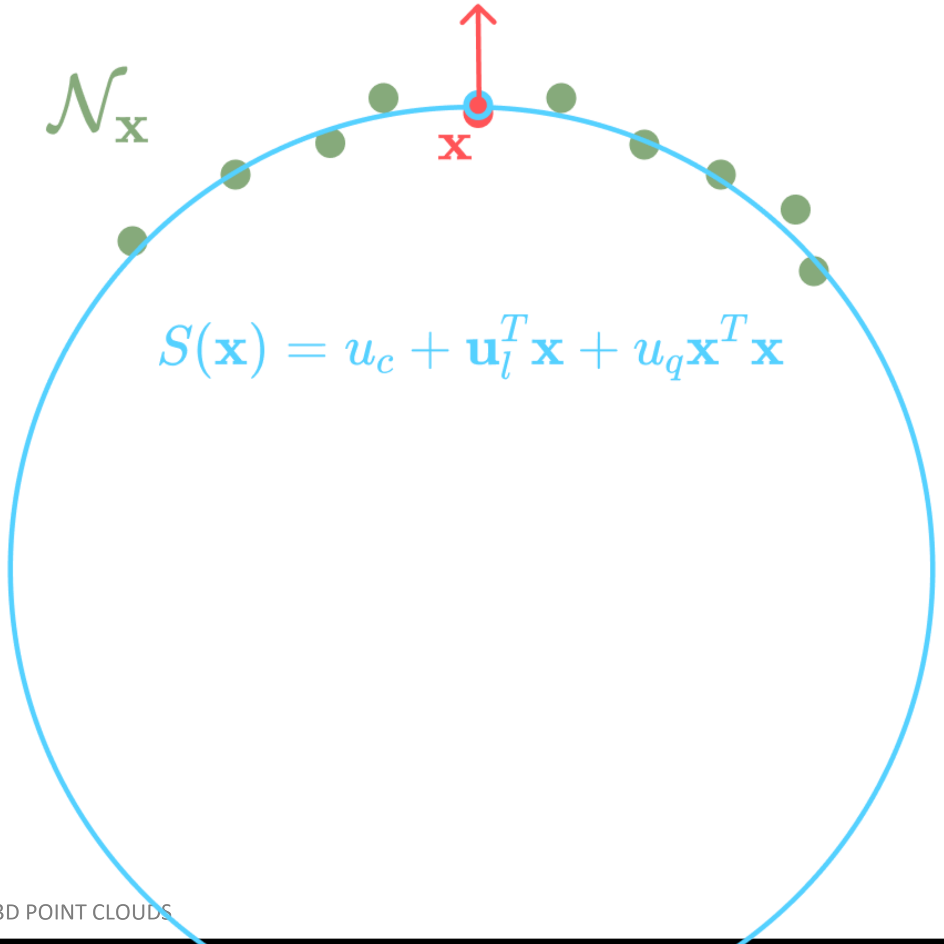
Methods – APSS [GG07]

- Estimate
  - Mean curvature H
- Requirements
  - Input normals
- Other sphere fitting variants
  - 3D positions only [Pra87]
  - non-oriented normals [CGBG13]



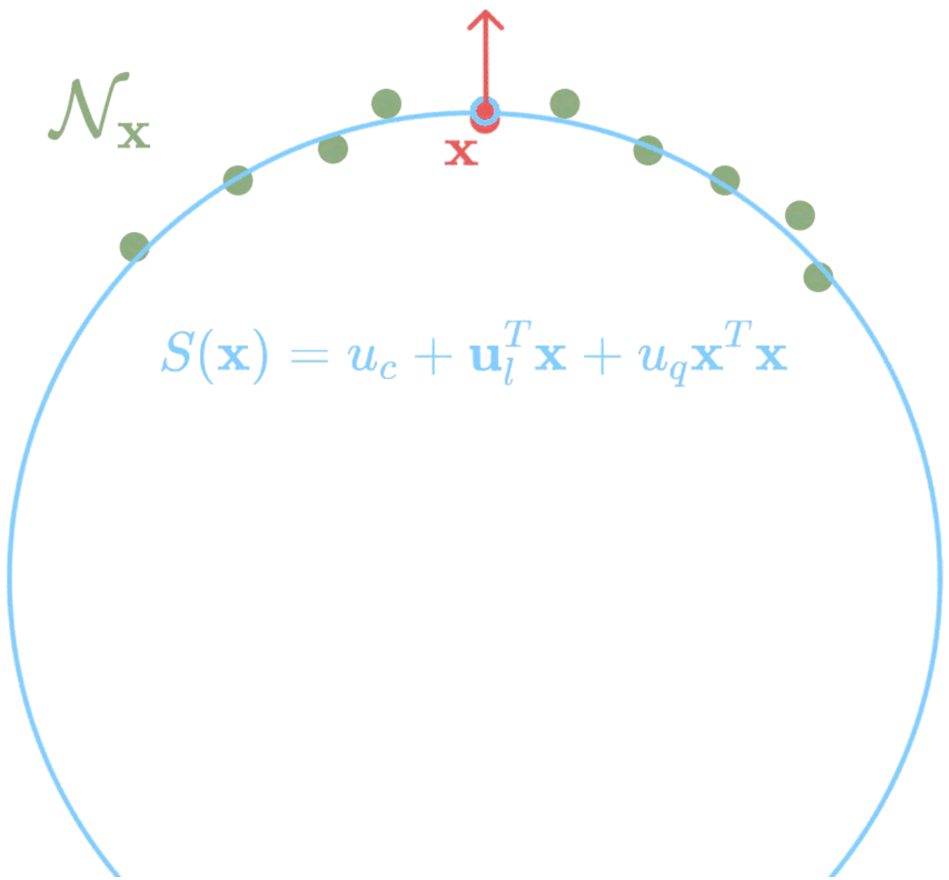
Methods – ASO [LCBM21]

- Extension to the APSS for Weingarten Map estimation



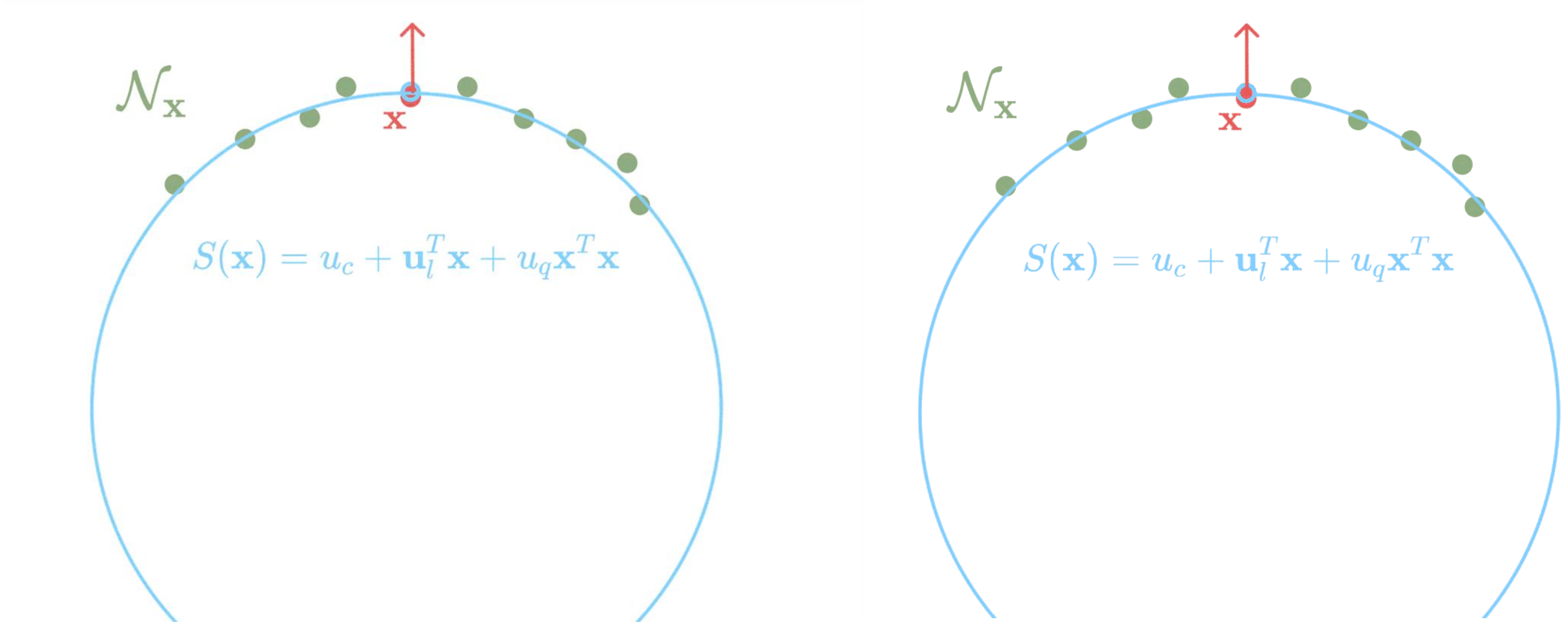
Methods – ASO [LCBM21]

- Extension to the APSS for Weingarten Map estimation
- Weight kernel differentiation
  - Variations of the normals on the estimated surface

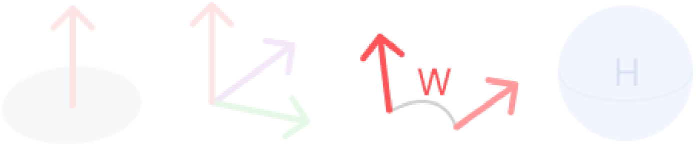


Methods – ASO [LCBM21]

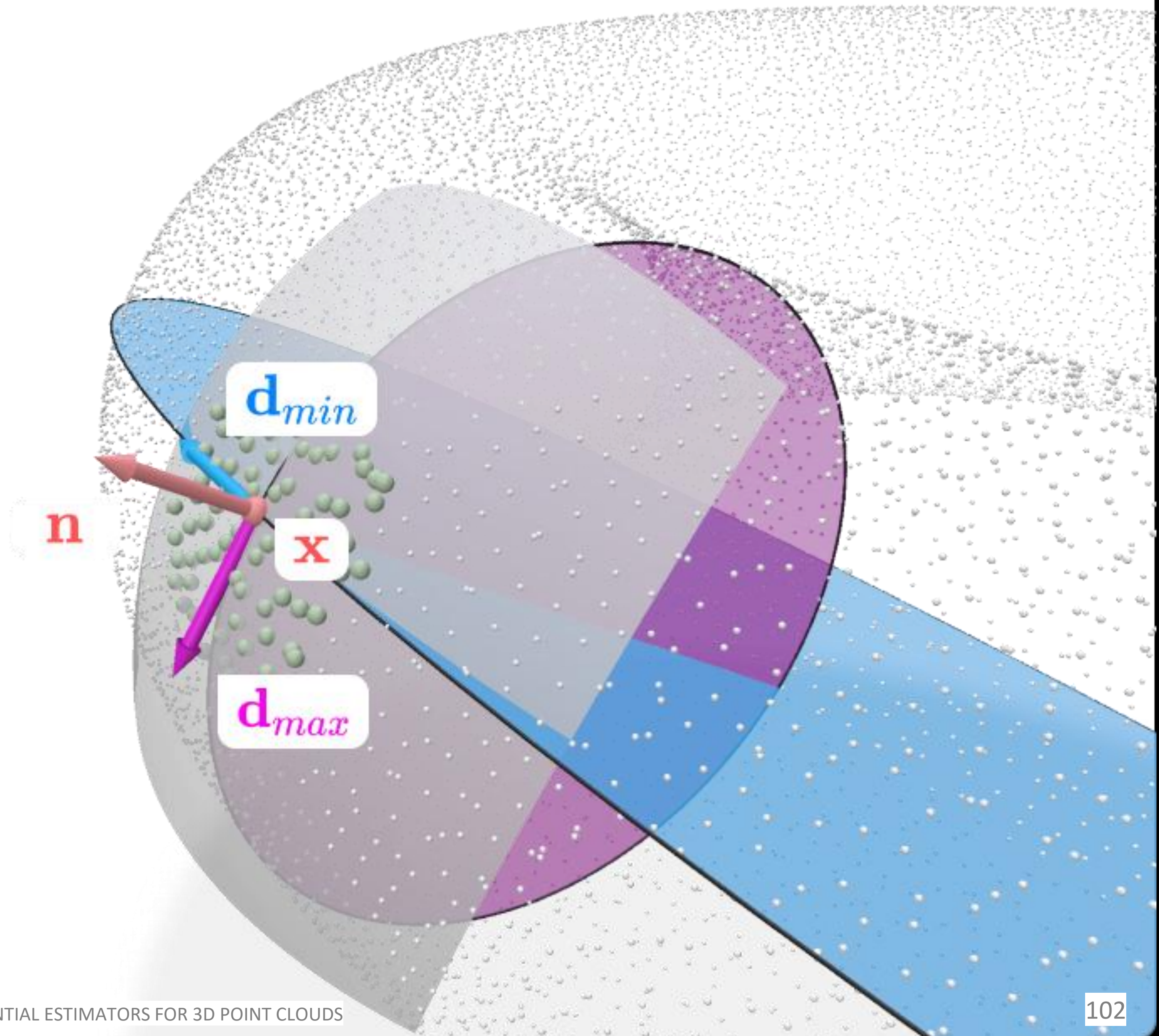
- Difference between APSS and ASO



Methods – ASO [LCBM21]

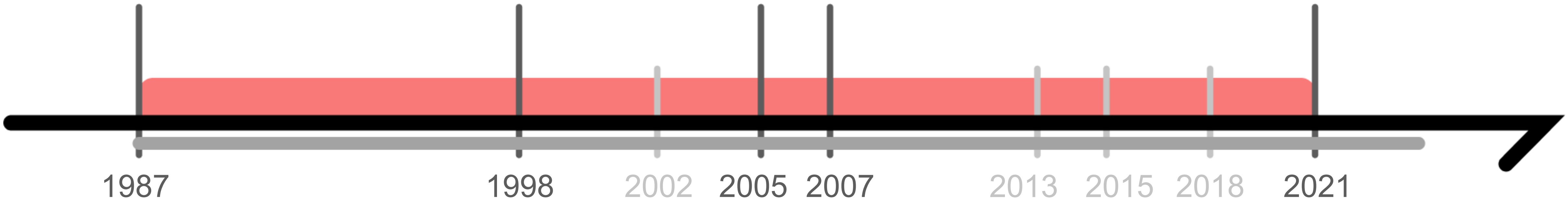


- Estimate
  - Weingarten Map
- Requirements
  - Oriented normals



Methods – Local surface models

- Analytic curvature estimation through geometric primitive fitting



Methods – Local surface models

Pros

- Stable
- « Gold standard » on perfect data (JetFitting)
- 3D methods avoid the two pass fitting (using tangent plane)

Cons

- 2.5D methods : Tangent plane - dependent
- The more the order of the primitive is, the more it tends to overfit the noise
- May be slower than direct point-based approaches

Methods – Measure theory approaches

- Geometric Measure Theory

Methods – AvgHexagram [LCL*23]

- Based on normal cycles [CSM03]
- Summations over locally generated triangles {i,j,k}

$$\mu_\tau^{(0)} = \frac{1}{2}\,\bar{\mathbf{n}} \cdot \left((\mathbf{p}_j - \mathbf{p}_i) \times (\mathbf{p}_k - \mathbf{p}_i) \right),$$

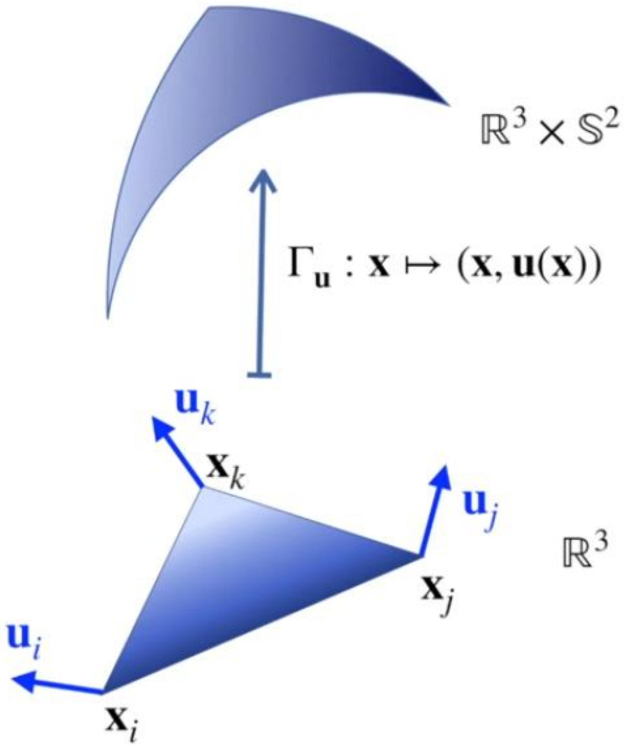
$$\mu_\tau^{(1)} = \frac{1}{2}\,\bar{\mathbf{n}} \cdot \left((\mathbf{n}_k - \mathbf{n}_j) \times \mathbf{p}_i + (\mathbf{n}_i - \mathbf{n}_k) \times \mathbf{p}_j + (\mathbf{n}_j - \mathbf{n}_i) \times \mathbf{p}_k \right),$$

$$\mu_\tau^{(2)} = \frac{1}{2}\,\mathbf{n}_i \cdot (\mathbf{n}_j \times \mathbf{n}_k),$$

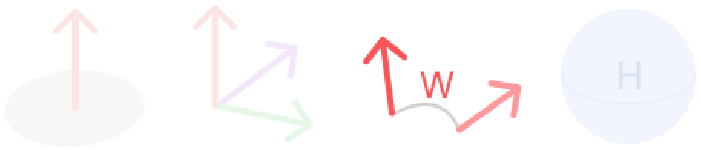
$$\mu_\tau^{(XY)} = \frac{1}{2}\,\bar{\mathbf{n}} \cdot \left((\mathbf{Y} \cdot (\mathbf{n}_k - \mathbf{n}_i))\mathbf{X} \times (\mathbf{p}_j - \mathbf{p}_i) - (\mathbf{Y} \cdot (\mathbf{n}_j - \mathbf{n}_i))\mathbf{X} \times (\mathbf{p}_k - \mathbf{p}_i) \right),$$

$$\bar{\mathrm{H}} = \sum_{\tau \in \mathrm{T}} \mu_\tau^{(1)} \Big/ \sum_{\tau \in \mathrm{T}} \mu_\tau^{(0)},$$

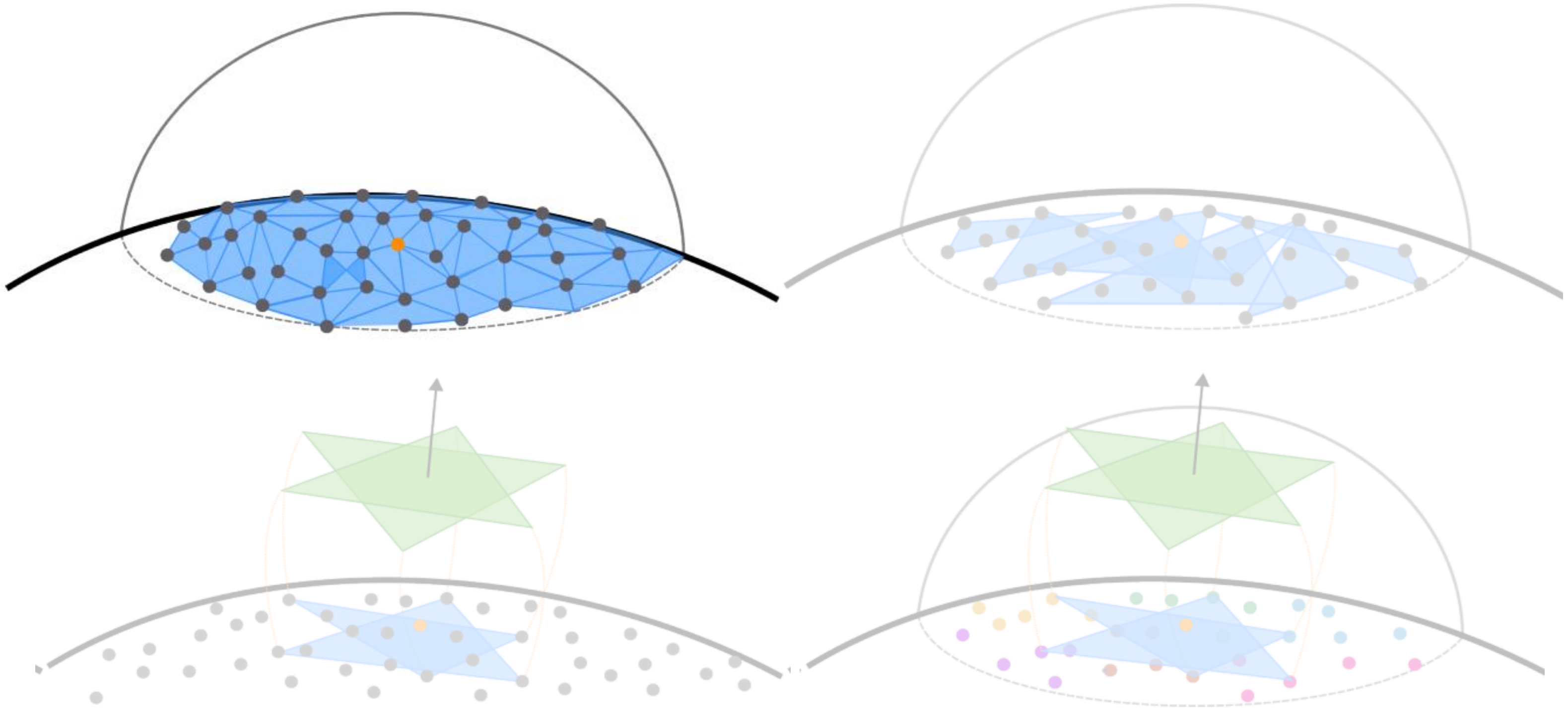
$$\bar{\mathrm{K}} = \sum_{\tau \in \mathrm{T}} \mu_\tau^{(2)} \Big/ \sum_{\tau \in \mathrm{T}} \mu_\tau^{(0)},$$



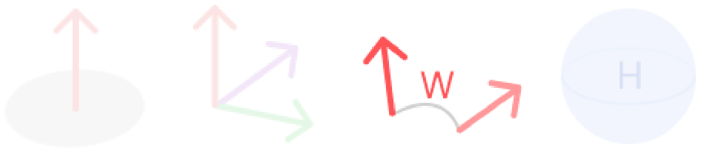
Methods – AvgHexagram [LCL*23]



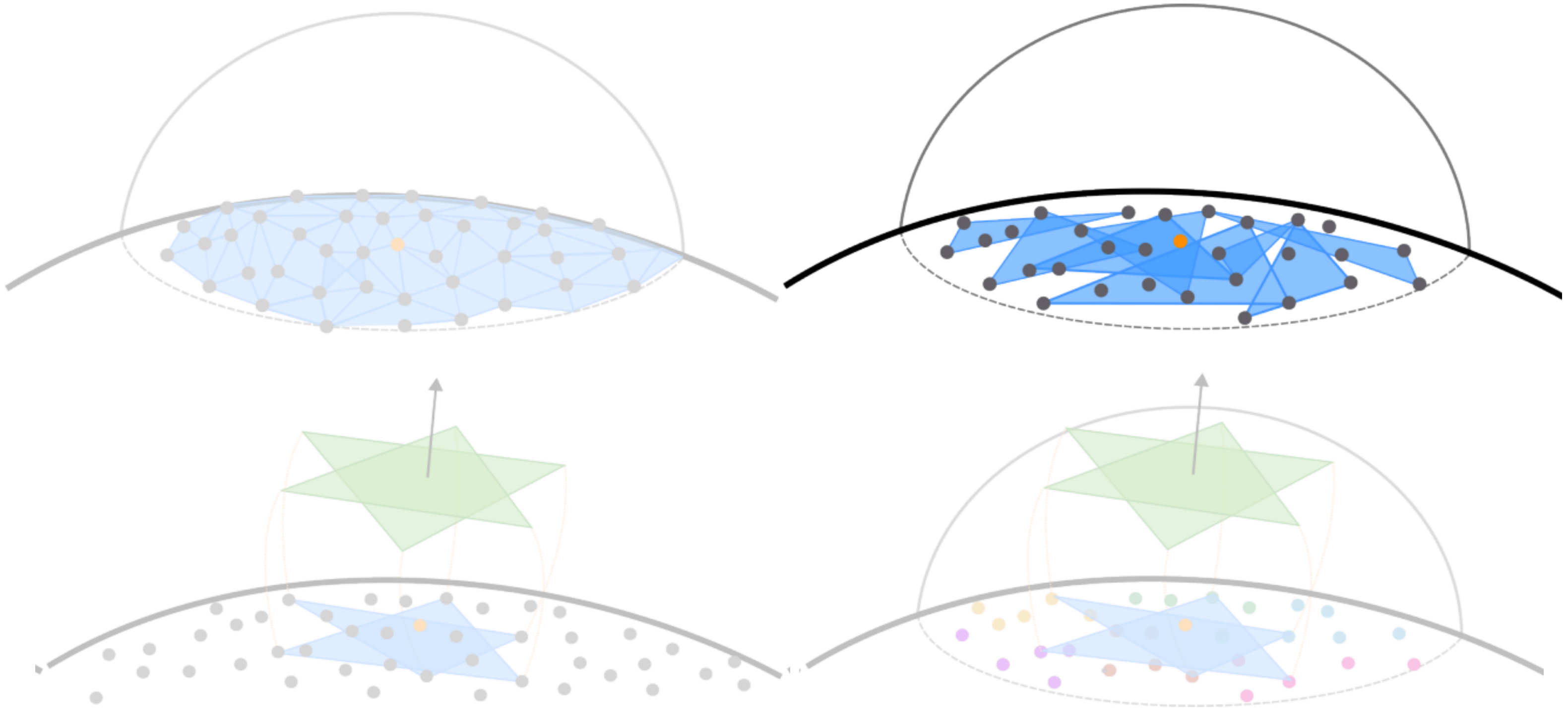
- Generation of the triangles in different ways



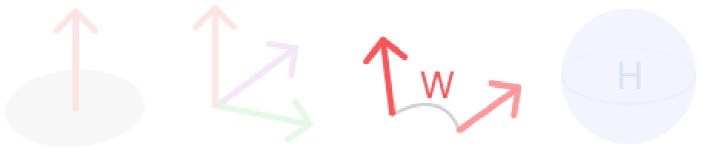
Methods – AvgHexagram [LCL*23]



- Generation of the triangles in different ways



Methods – AvgHexagram [LCL*23]



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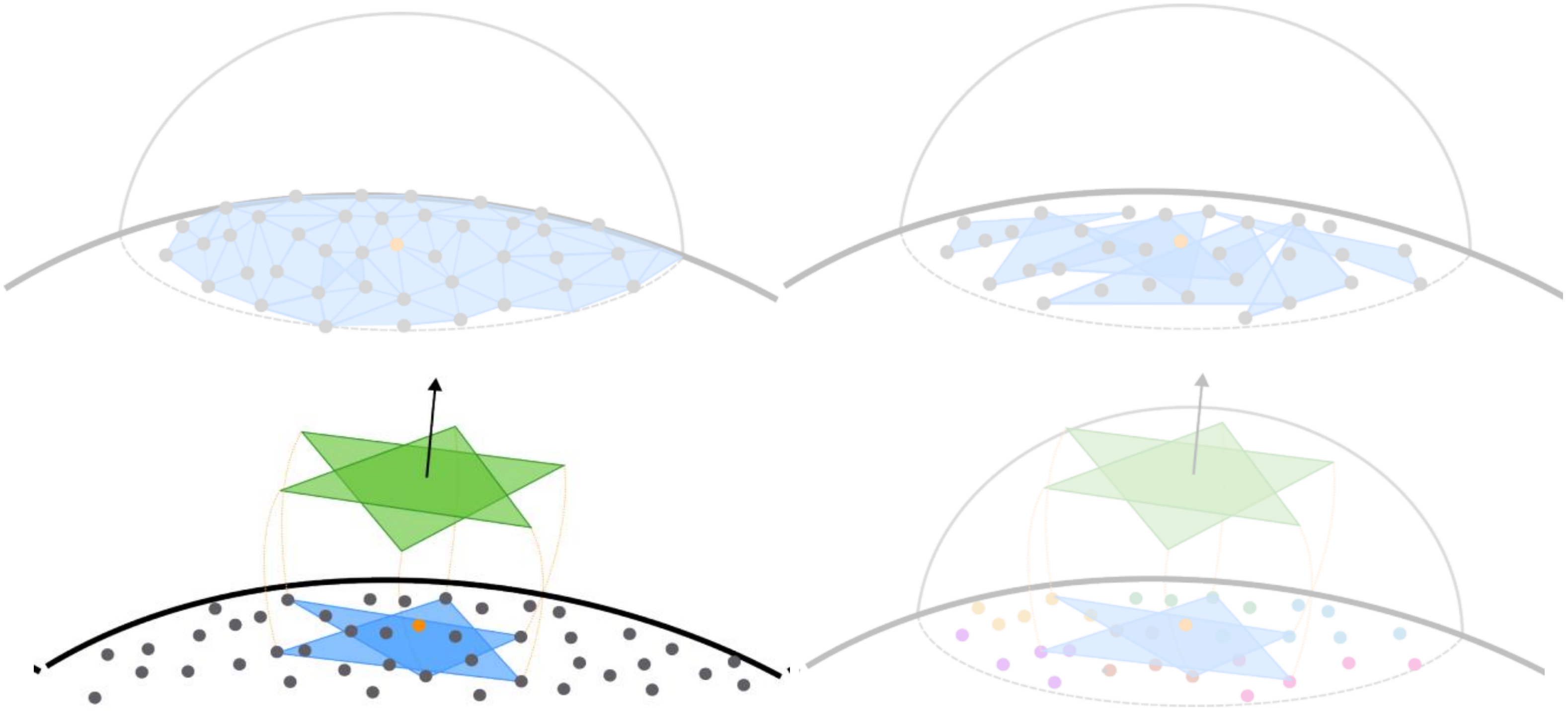
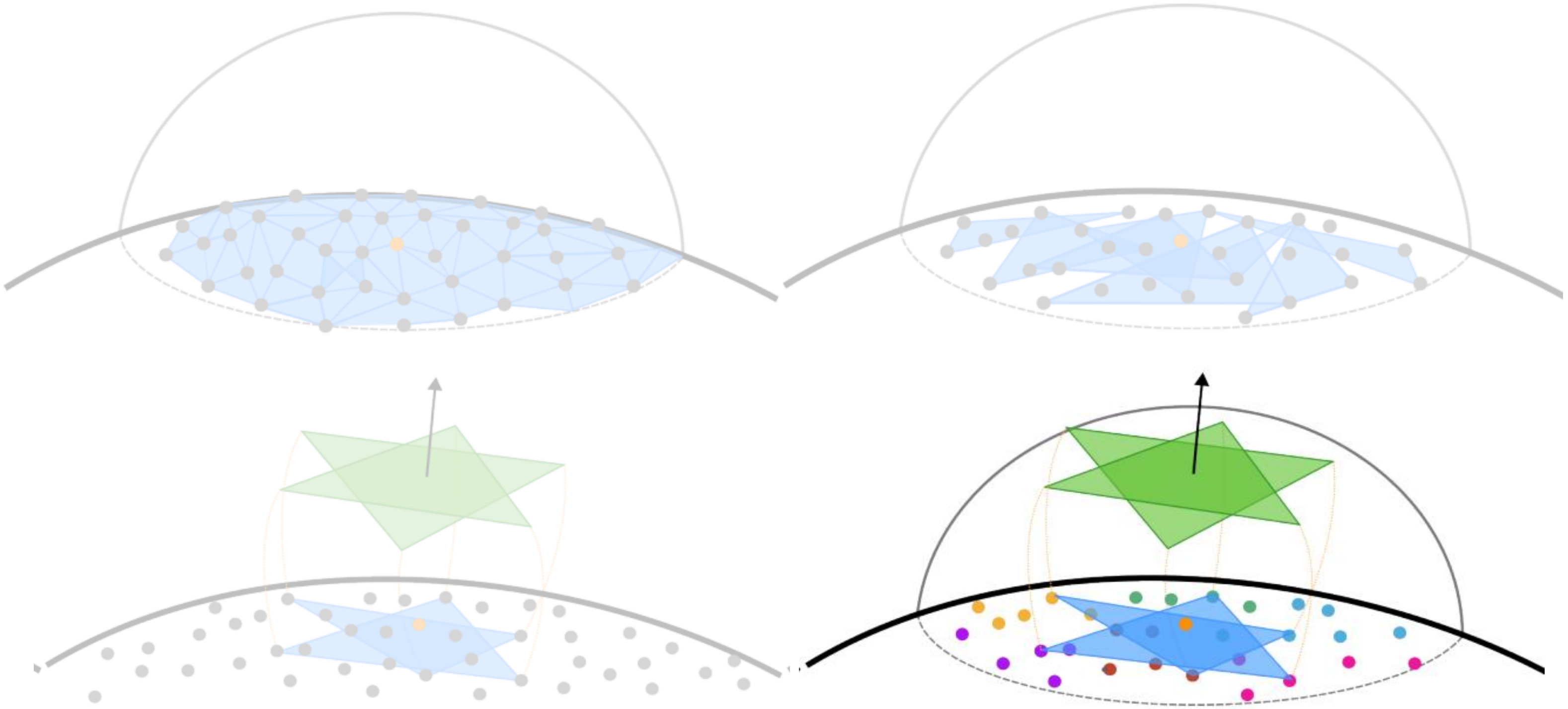


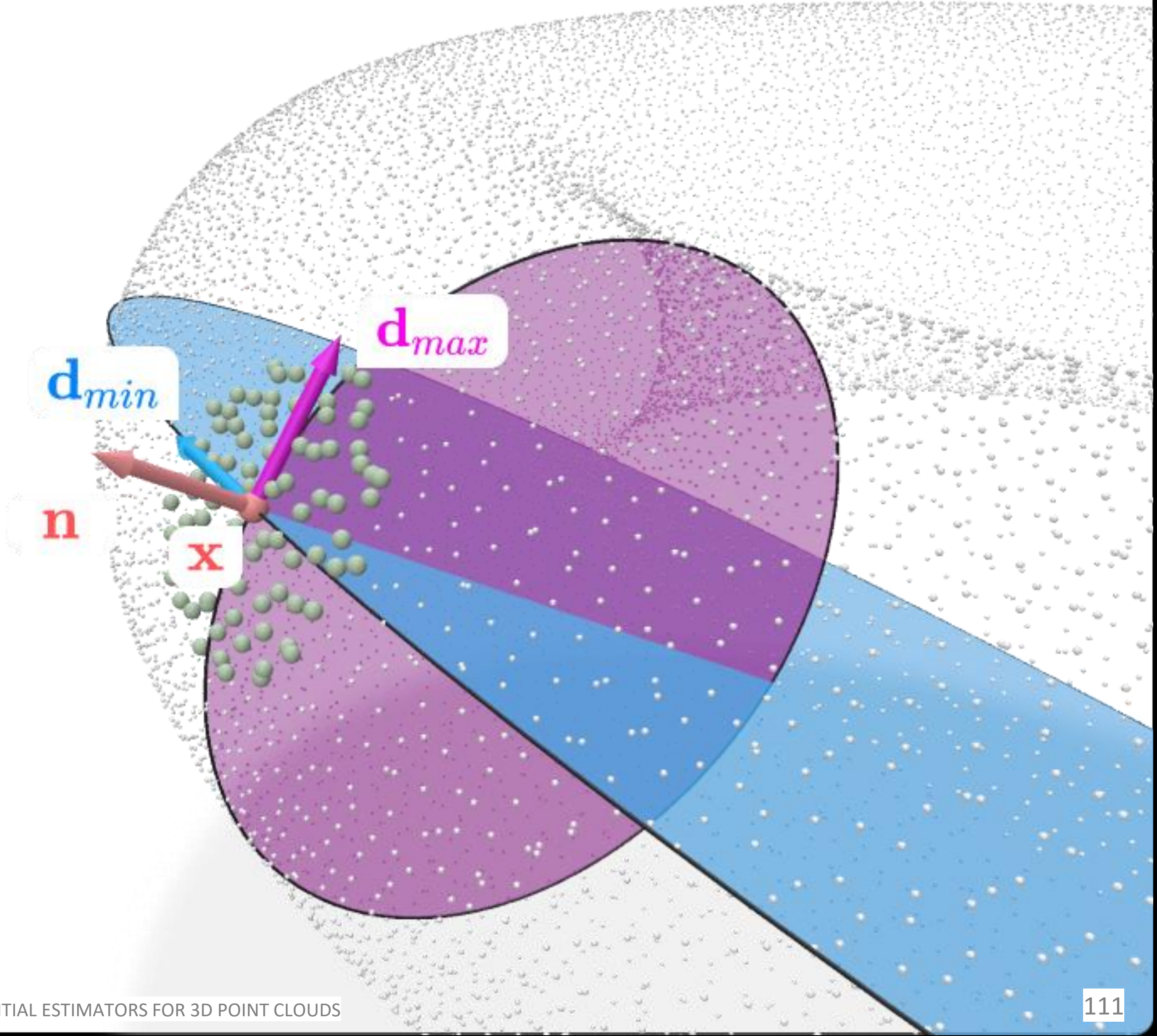
Methods – AvgHexagram [LCL*23]

- Generation of the triangles in different ways



Methods – AvgHexagram [LCL*23]

- Estimate
  - Weingarten Map
- Requirements
  - Oriented normals



Methods – Measure theory approaches

- Define curvatures as measures on the bundle {x,n}

Methods – Measure theory approaches

Pros

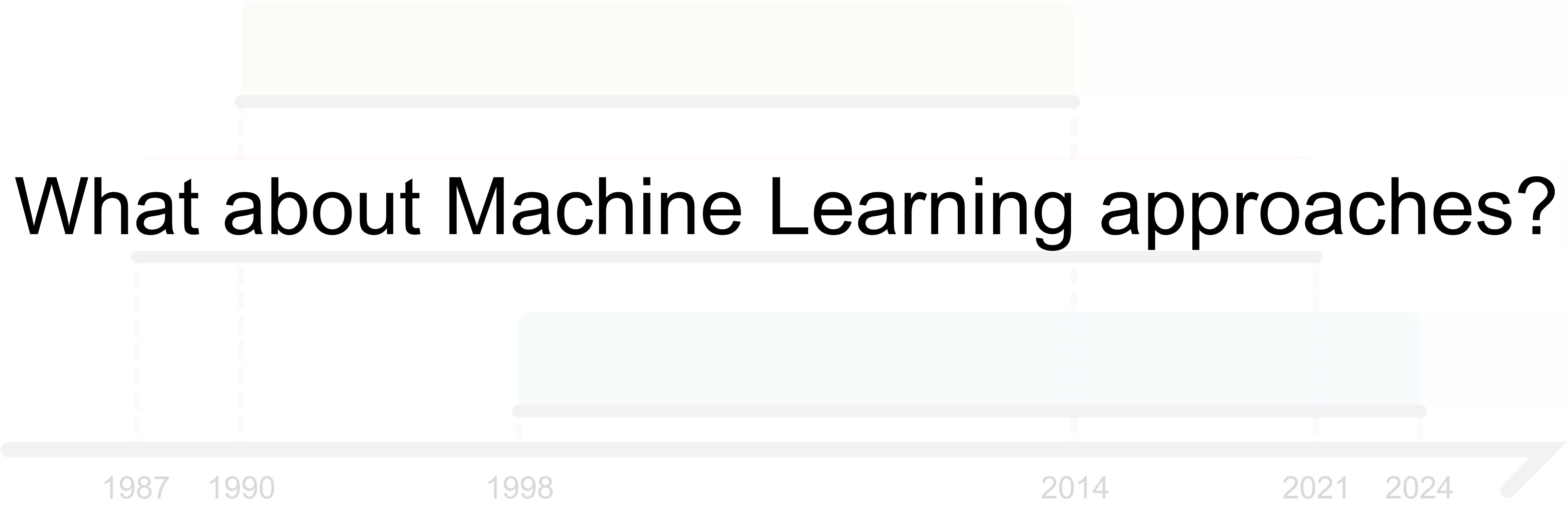
- Theoretical guarantees (convergence and stability)
- Robustness to noise position
- AvgHexagram time-efficient

Cons

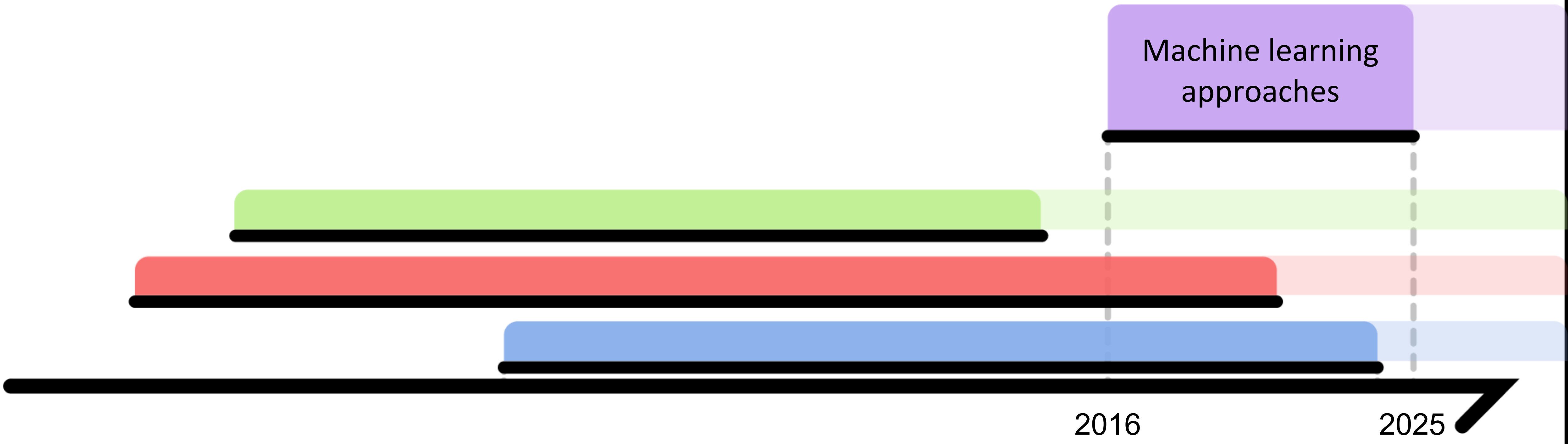
- Normal dependent

Methods – Overview

What about Machine Learning approaches?



Methods – Overview



Methods – Machine learning approaches

- Regression-based
  - Learn features on the point cloud directly

  [BM16, QSMG17, GKOM18, ZNZ*25]

Methods – Machine learning approaches

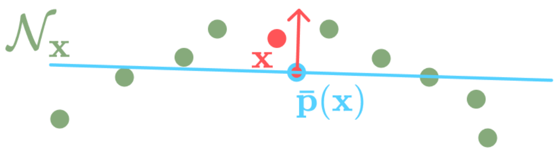
- Weight estimation
  - Offset points [ZLD*21, LZW*22, ZJW*23]
  - Weights [LOM20, BSG20, ZLD*21, ZJW*23]
  - Primitive fitting (plane or jet)

Methods – Machine learning approaches

- Local surface estimation
  - Differential RANSAC model [CZB*22]
  - Hypersurface fitting on the feature space [LLC*22, LFS*23]
  - Derivatives of implicit surface [LSY24]

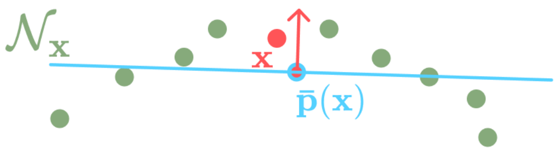
Methods – Machine learning approaches

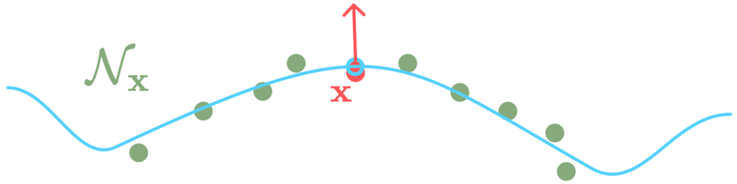
- Most of these are dedicated (but not limited) to normal estimation
- The better ones often rely on classical primitive fitting
  - Plane [LOM20]



Methods – Machine learning approaches

- Most of these are dedicated (but not limited) to normal estimation
- The better ones often rely on classical primitive fitting
  - Plane [LOM20]
  - JetFitting [BSG20, ZLD*21, ZJW*23]



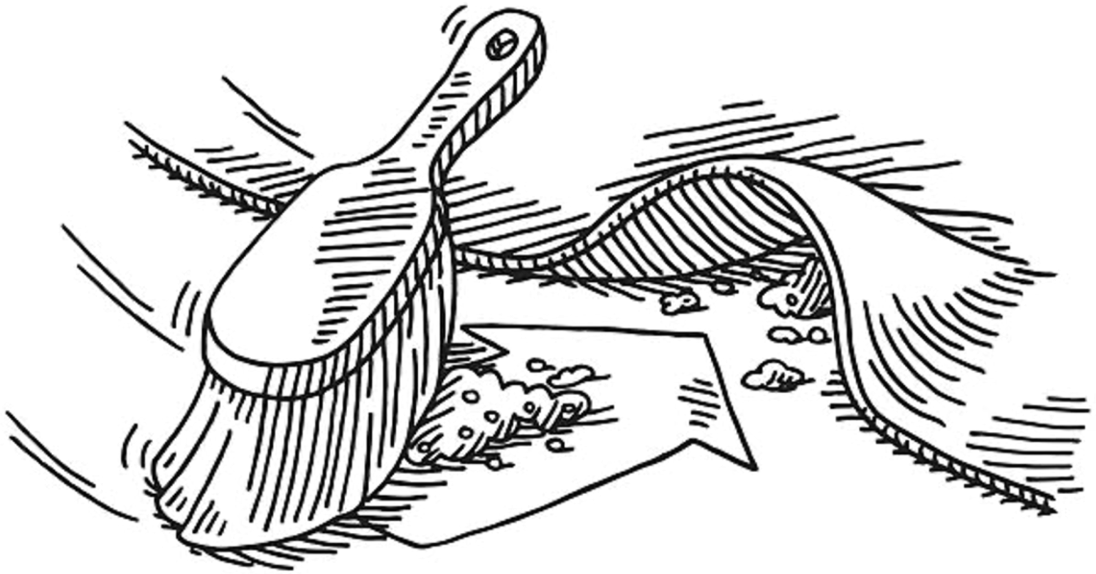


Methods – Machine learning approaches

- Most of these are dedicated (but not limited) to normal estimation
- The better ones often rely on classical primitive fitting
  - Plane [LOM20]
  - JetFitting [BSG20, ZLD*21, ZJW*23]
- Why choose these primitive instead of others?

QUESTION TIME

What’s under the rug?



What’s under the rug? – Weighting schemes

- Neighborhood weighting kernels

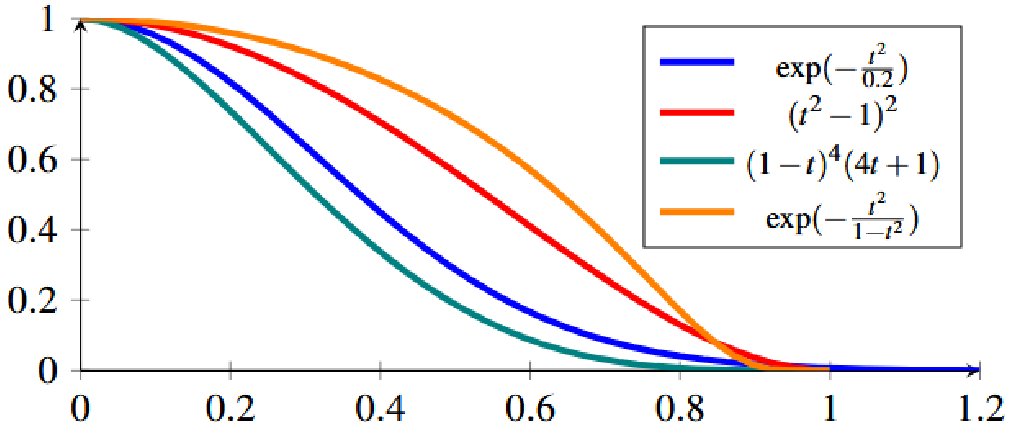
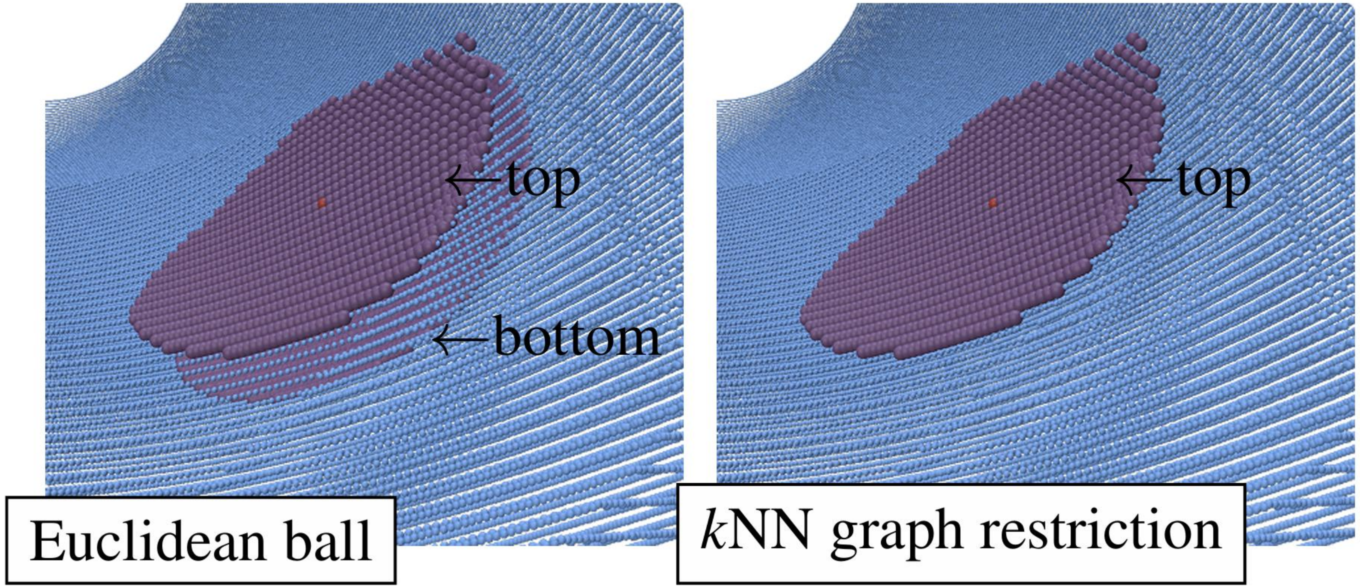


Figure 6: *Standard weighting kernels Ω: Gaussian, quartic, Wendland [Wen95], and bump [BLM22].*

What's under the rug? – Neighborhood collection

- Neighborhood weighting kernels
- Neighborhood collection



Benchmark

04

Benchmark

- Benchmark ecosystem
  - Datasets
  - Quantitative analysis tools
  - Qualitative analysis tools
  - Visualization
  - Website

[Link to the website]

Benchmark

- Benchmark ecosystem
  - Datasets → Open
  - Quantitative analysis tools → Open
  - Qualitative analysis tools → Open
  - Visualization → Open
  - Website → Open

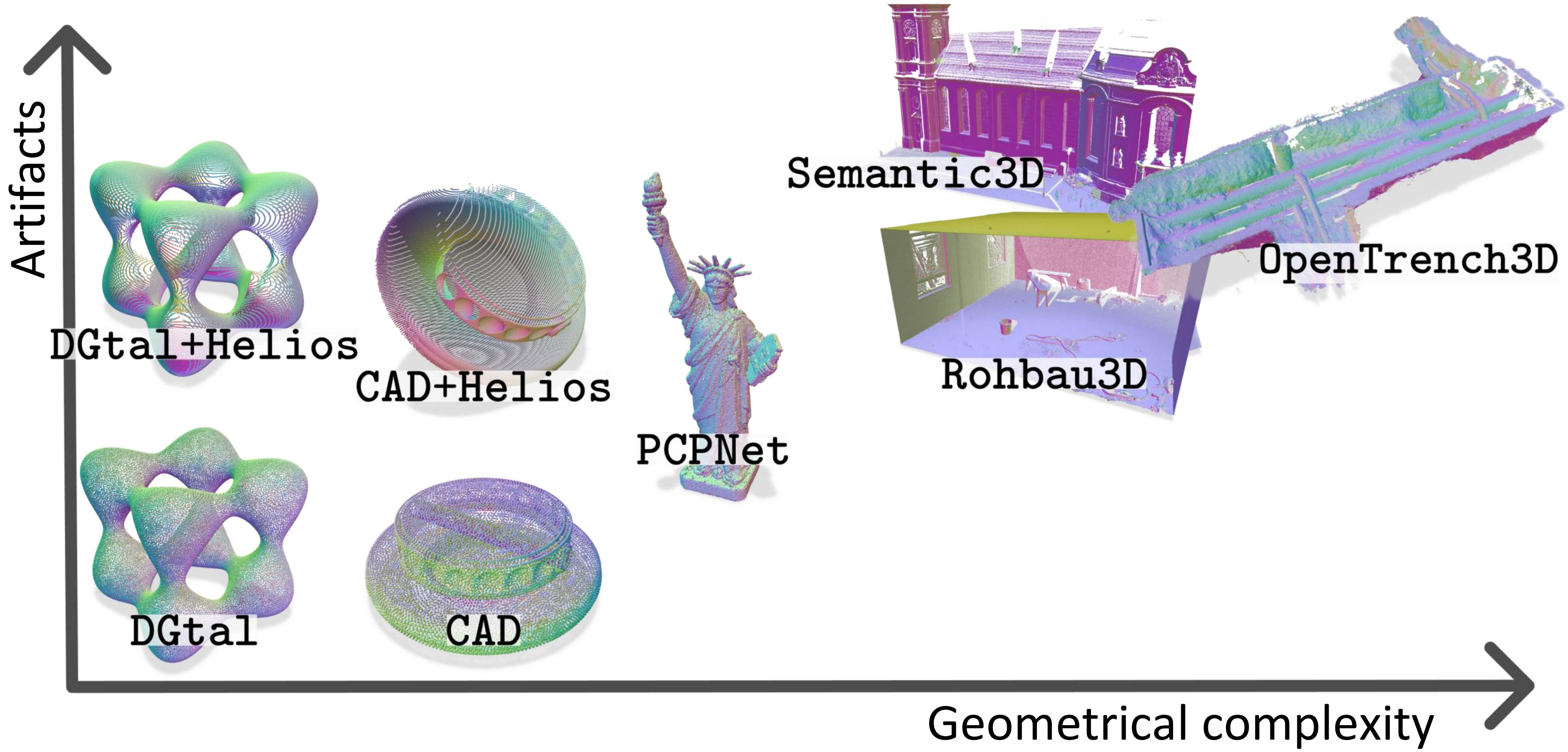
Link to the website

Benchmark

- Estimators
  - Ponca [MLG∗20] (also for kD-Tree)
  - CGAL [PC24] for JetFitting
  - Eigen for the maths
- Visualization tools
  - Polyscope
- Dataset generation
  - DGtal [DGt]
  - Helios++ [WEW*22] for LiDAR-like noise

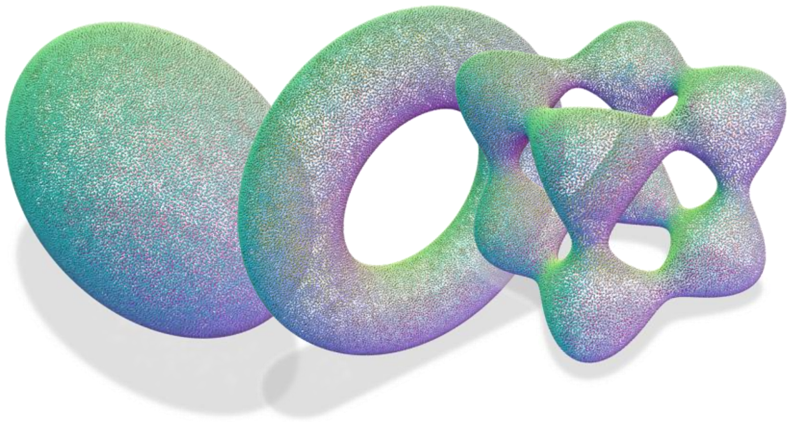
Link to the website

Benchmark - Datasets



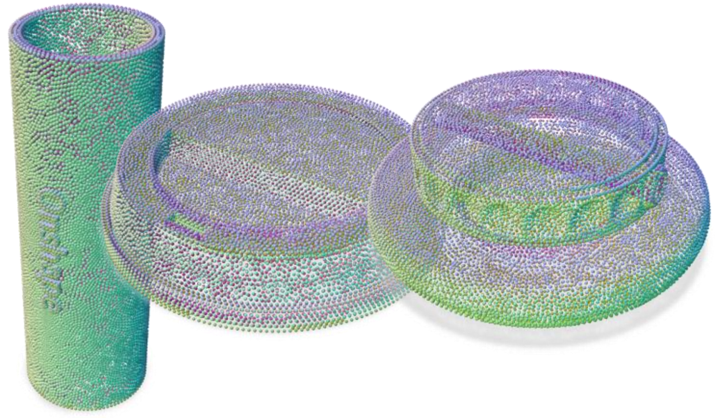
Datasets – DGtal

- Base of our benchmarking tool
- Samples of implicit polynomial surfaces
  - Generated with DGtal [DGt]
- Ground-truth
  - Weingarten maps



Datasets – CAD

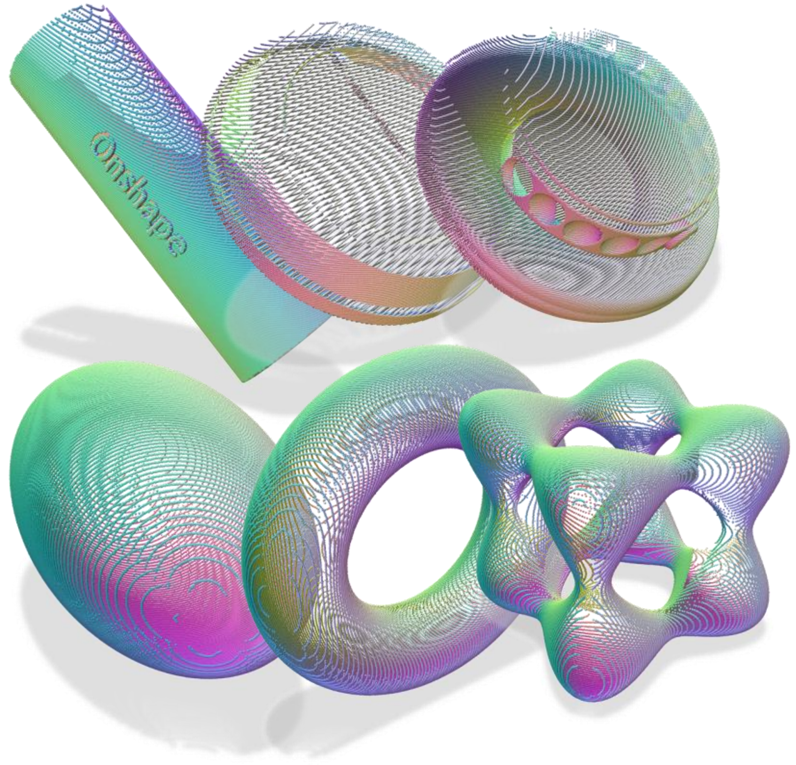
- Coming from the ABC dataset [KMJ*19]
- Sampled from the meshes
  - Poisson disk sampling [Yuk15]
- Ground-truth
  - Normals



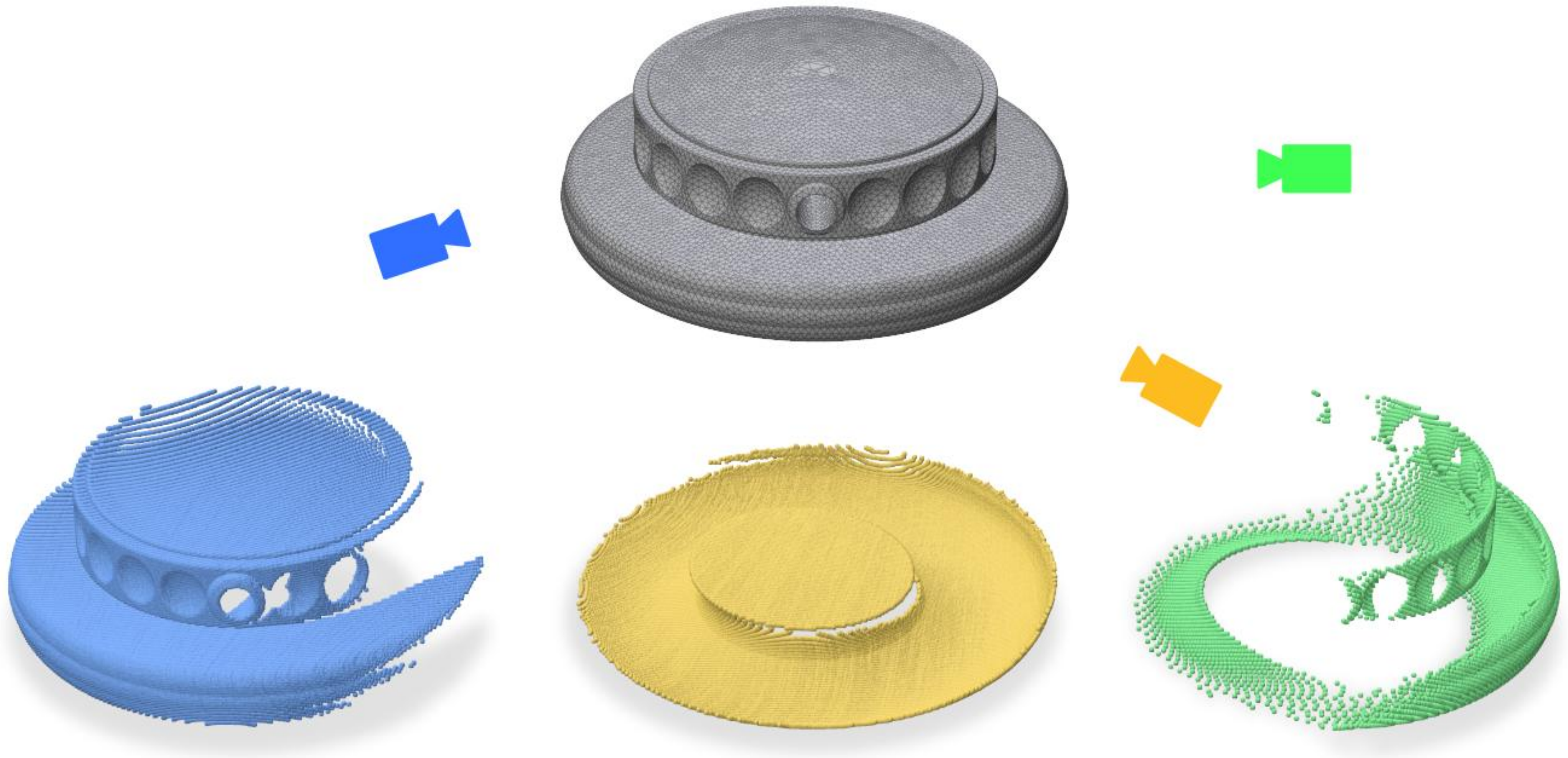
Datasets – Helios

- LiDAR-like density variation
  - Helios++ [WEW*22]

- Over the DGtal dataset
- Over the ABC dataset



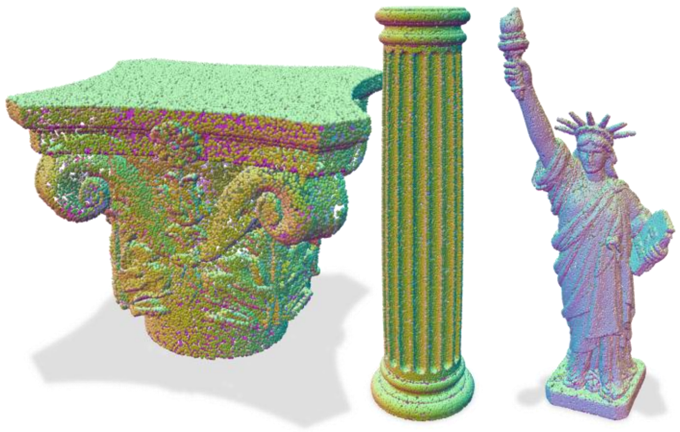
Datasets – Helios



Datasets – PCPNet

- Widely used in deep learning
- Test set of PCPNet Dataset

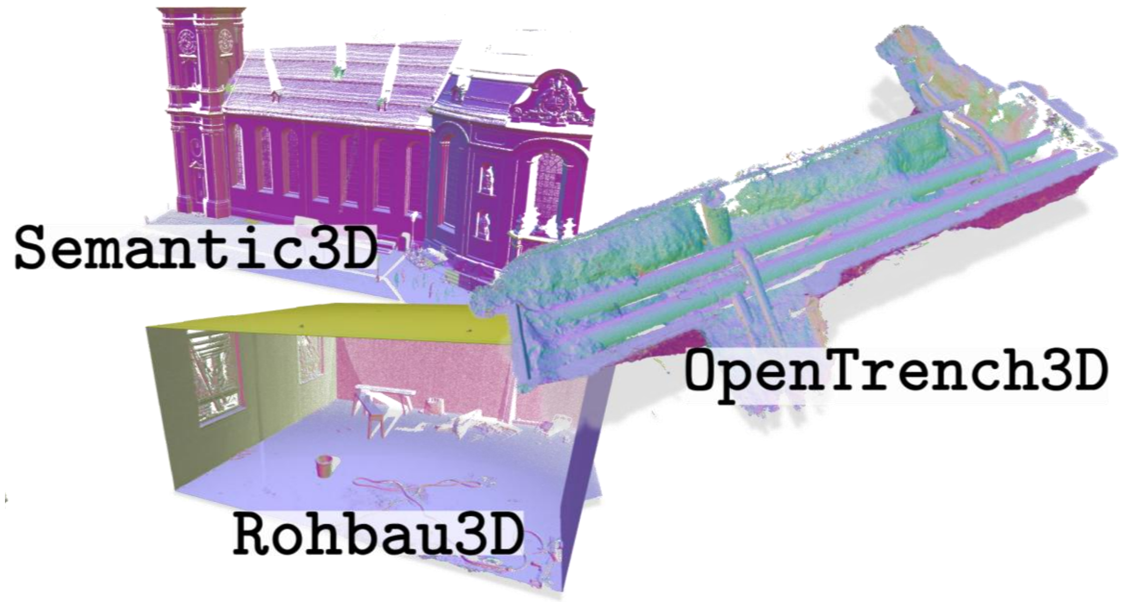
- Ground-truth
  - (mesh-estimated) curvatures
  - Normals



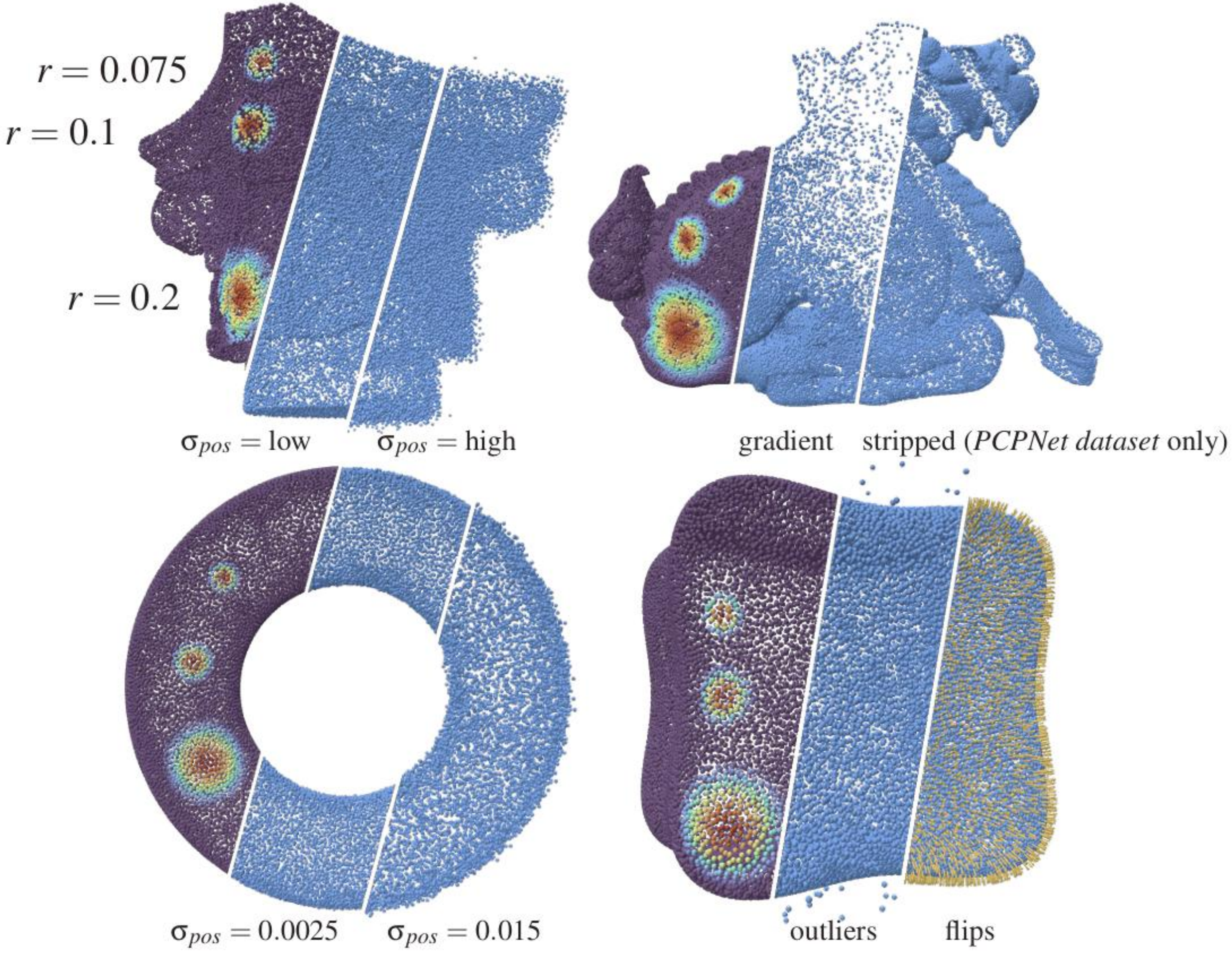
Datasets – Real World

- Used for qualitative evaluation
- Rohbau3D [RB25]
- OpenTrench3D [HJP∗24]
- Semantic3D [HSL∗17]

- PCA-estimated normals
  - Z-oriented



Datasets – Perturbations



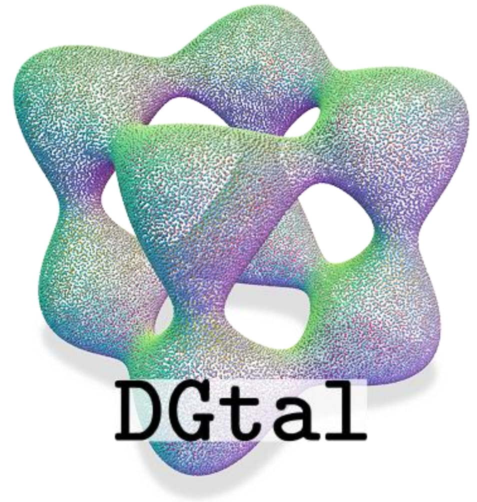
Benchmark – Evaluation

- Metrics
  - RMSE
  - RMS Angular E

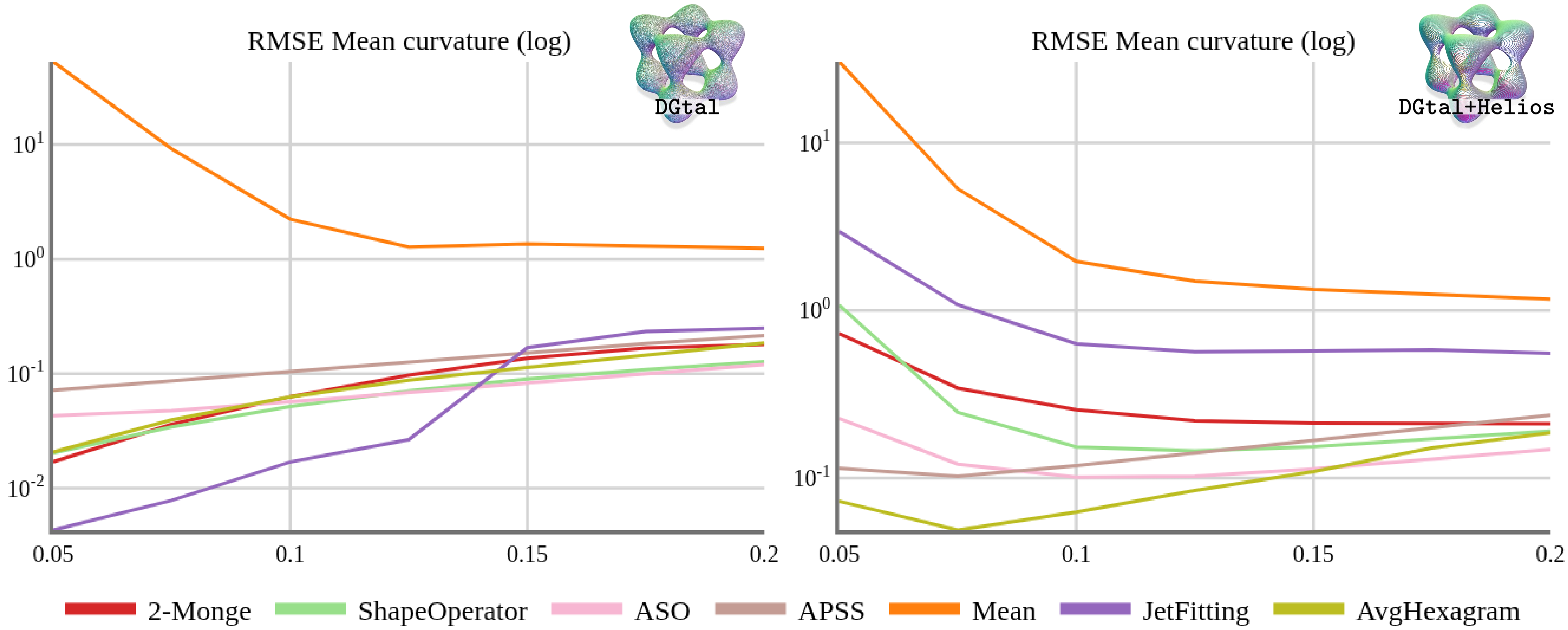
Benchmark – Settings

- DGtal dataset (left)
  - Clean data (no noise)
  - 50 000pts per shape

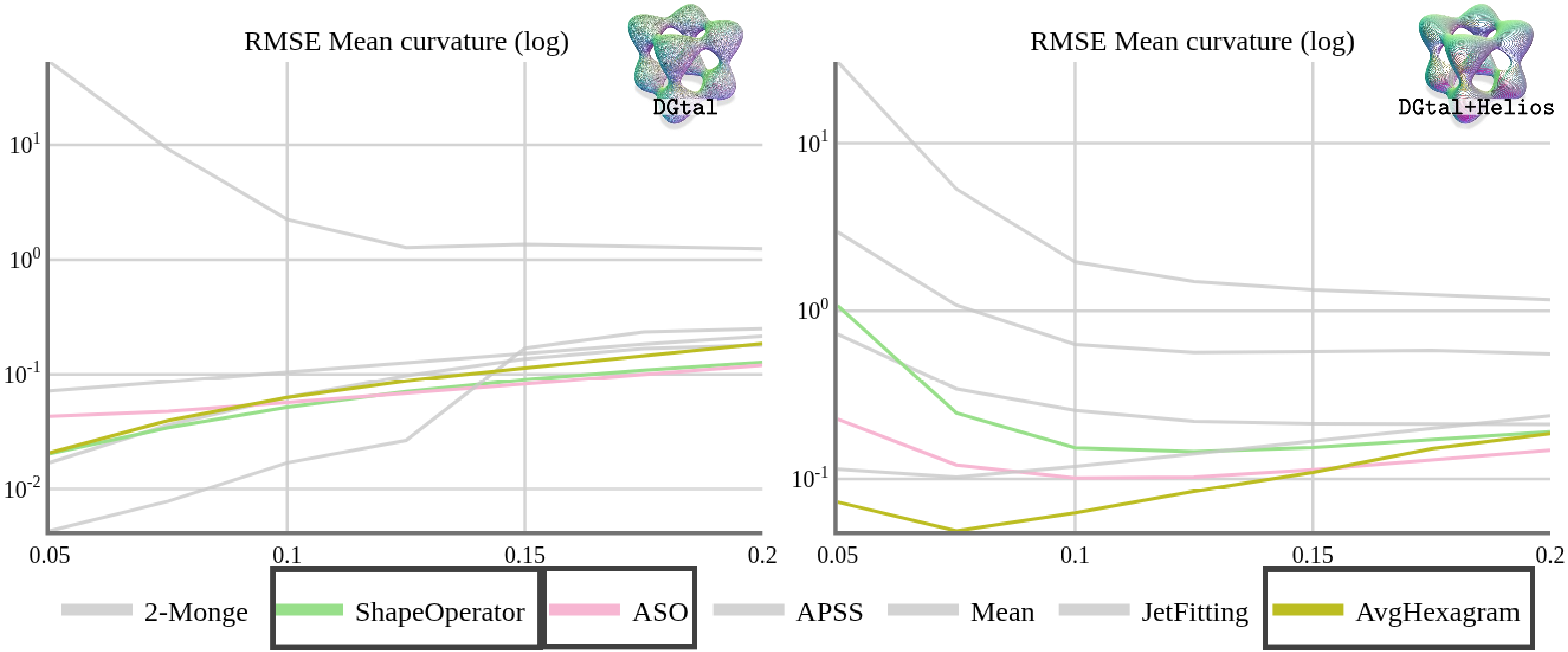
- DGtal + Helios dataset (right)
  - No added noise
  - Density variation with Helios



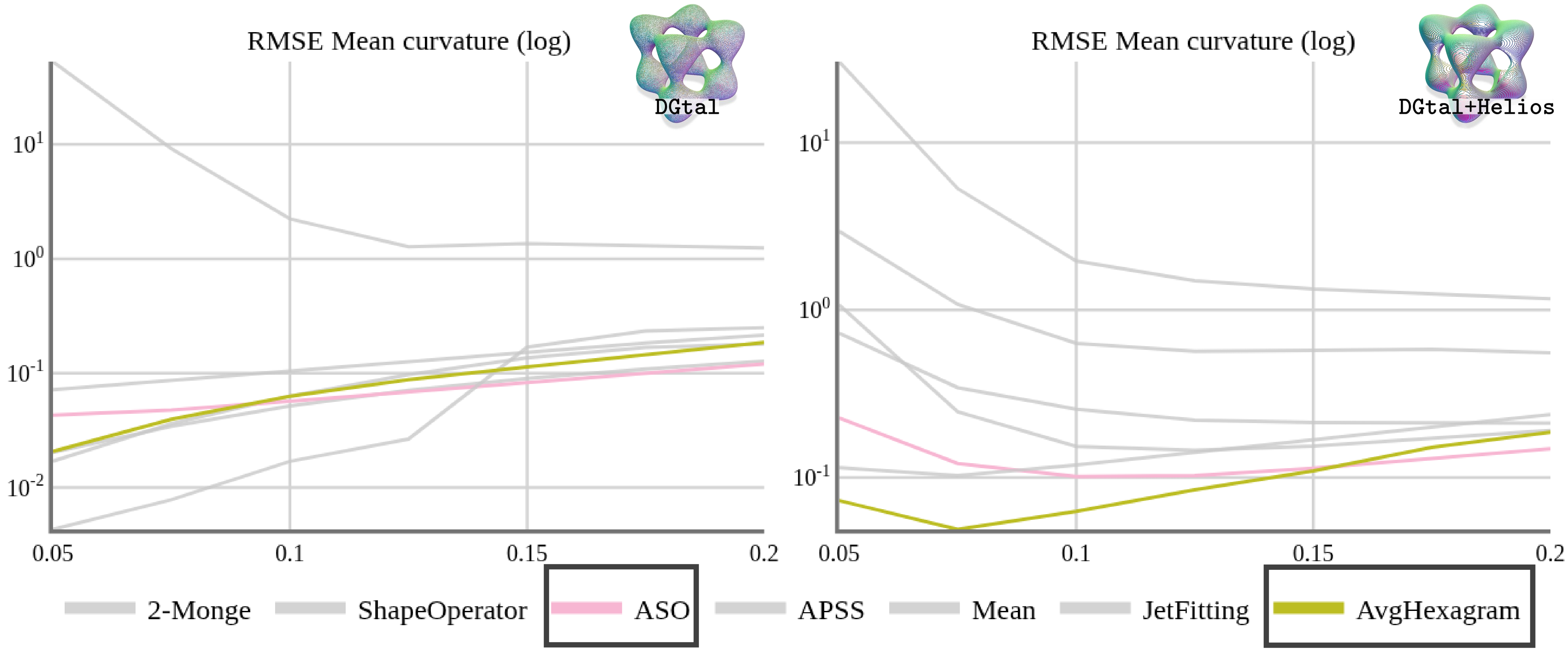
Results and take away messages



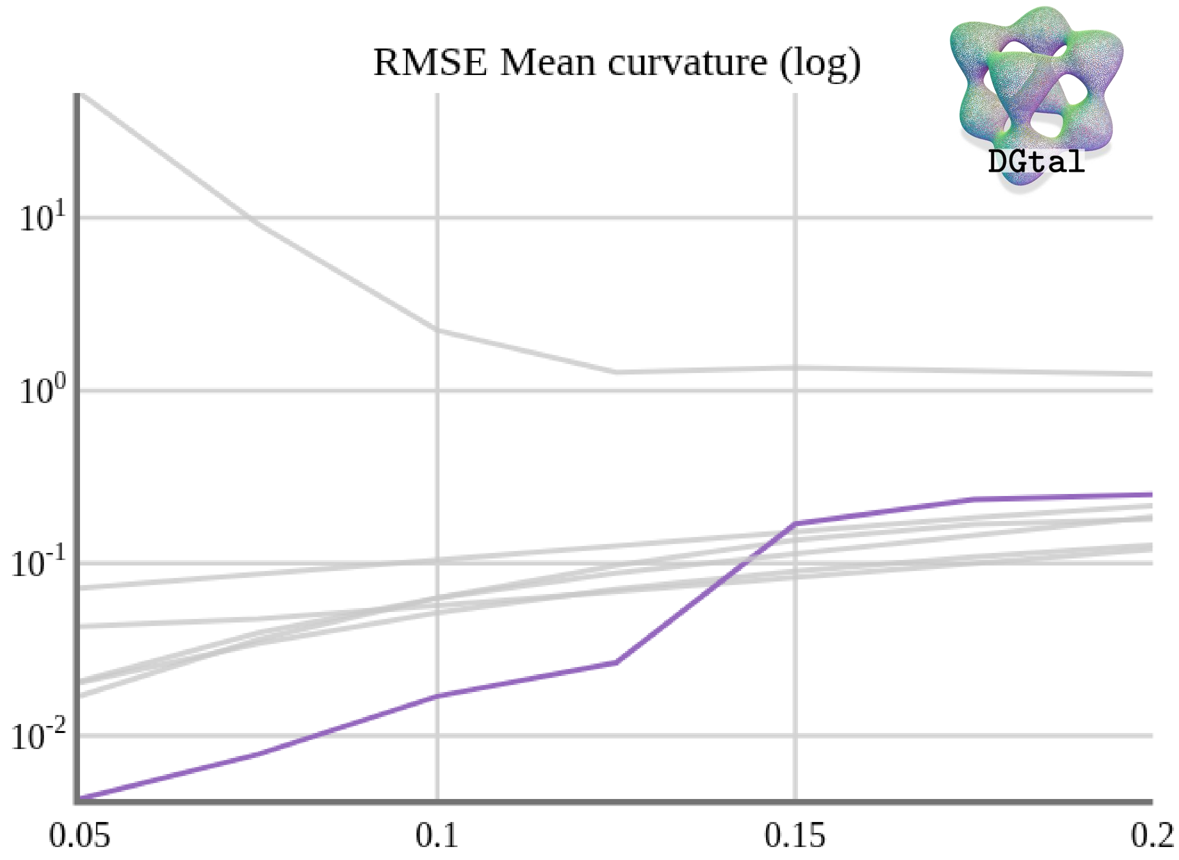
Results and take away messages

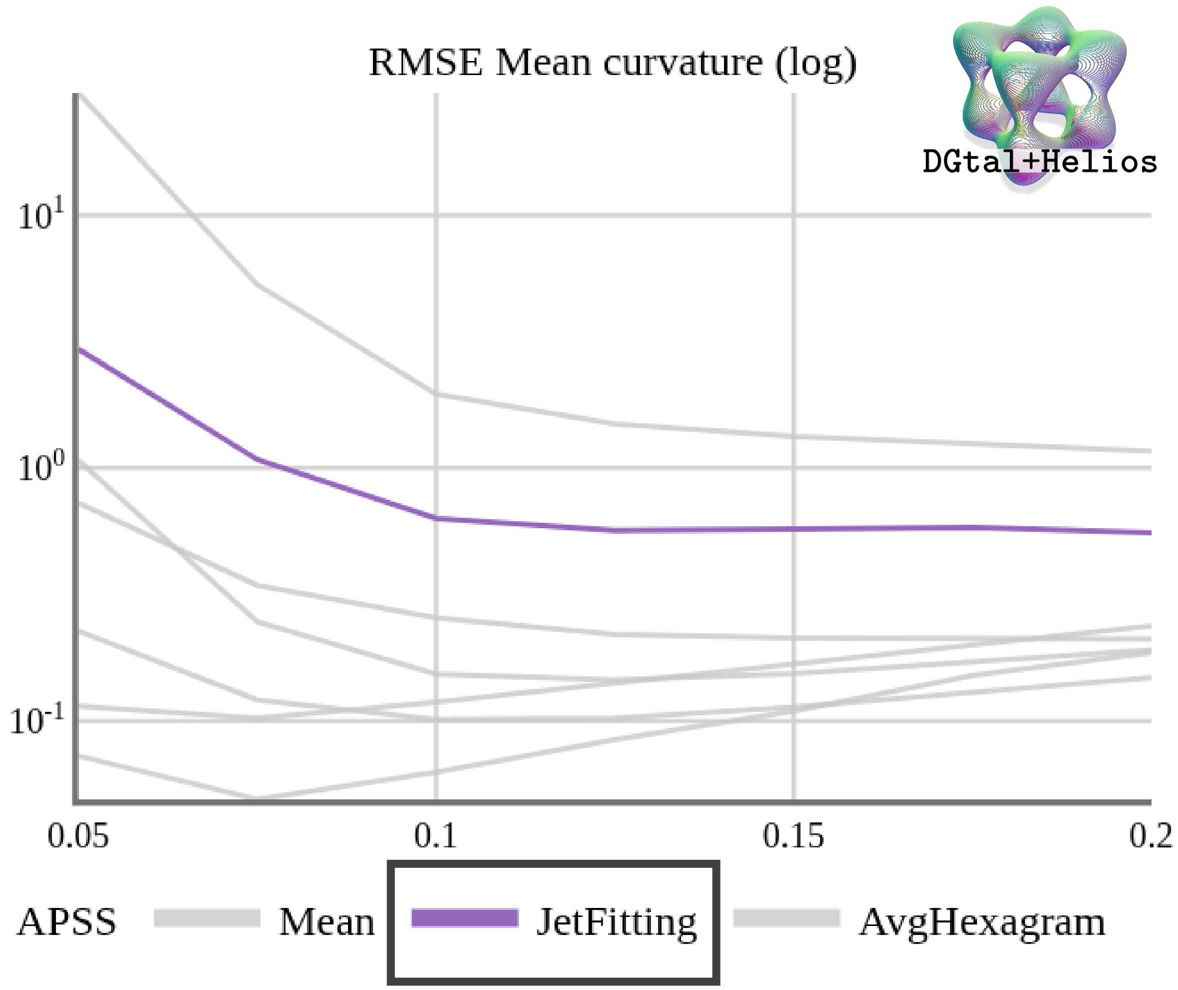


Results and take away messages

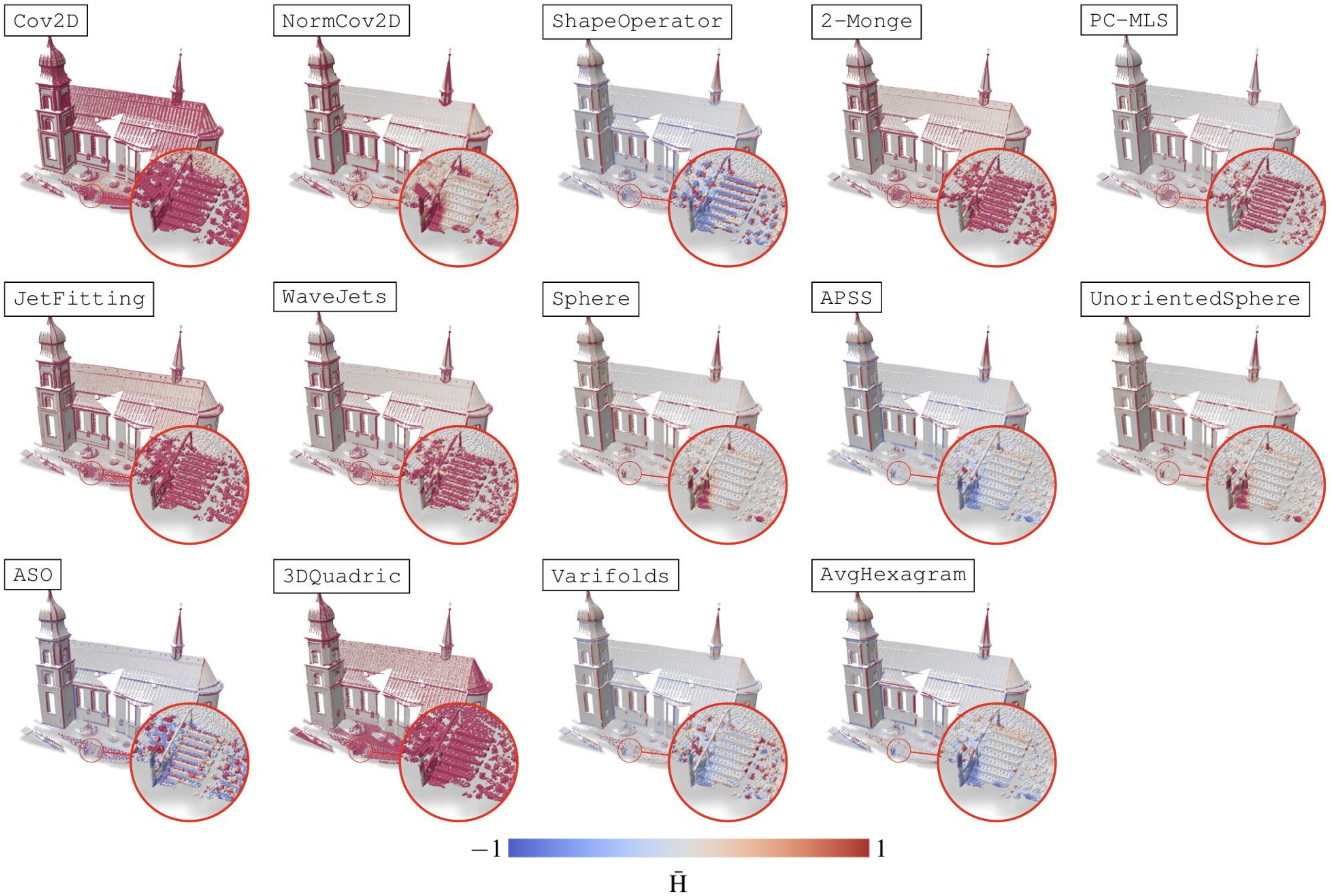


Results and take away messages

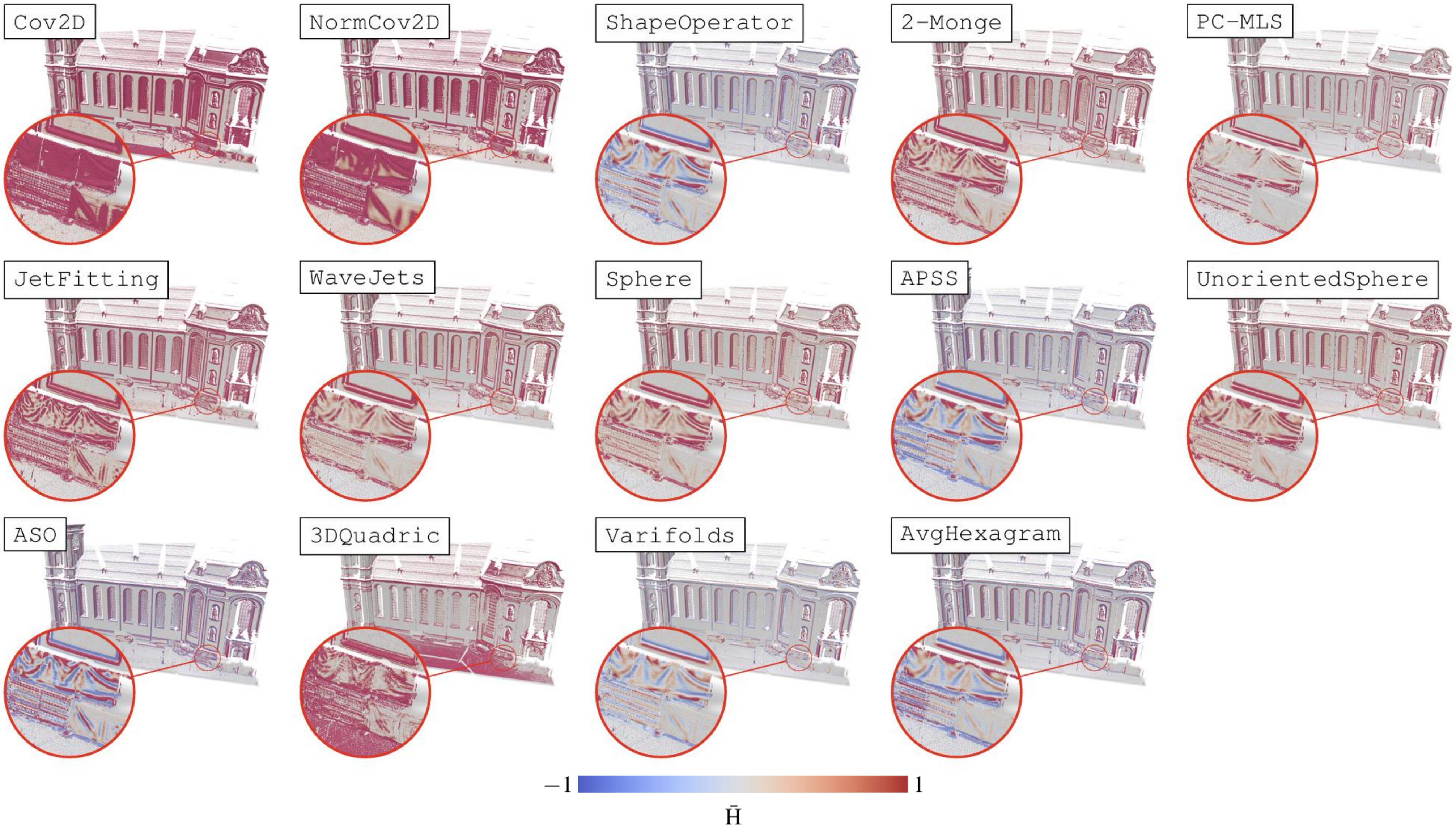




Results and take away messages



Results and take away messages



Future works

05

Future works

- Deeper analysis of the dependence on normals and orientation
- Unified benchmark including deep learning and ‘classical’ approaches
- Include prior knowledge of sensors
- Higher order differential properties

Conclusion

06

Summary

- Survey of differential estimators on 3d point cloud

  Direct point-based approaches
  Local surface models
  Measure theory approaches
  Machine learning approaches

- Open source benchmark
  - Several datasets
  - Ecosystem (data analysis, evaluation, website)
- Results
  - No « best » method
  - Depends on radius, type of surface, differential property, input requirements ...

Thanks!

storm-irit.github.io/pcloud-differential-estimation-benchmark-website

Thanks!

Léo is looking for a post doc!

Bibliography

[BKP∗10] BOTSCH M., KOBBELT L., PAULY M., ALLIEZ P., LEVY B.: Polygon Mesh Processing. A K Peters, 2010. doi:10.1201/ b10688.

[VVP∗16] VÁŠA L., VANˇ Eˇ CEK P., PRANTL M., SKORKOVSKÁ V., MARTÍNEK P., KOLINGEROVÁ I.: Mesh Statistics for Robust Curvature Estimation. Computer Graphics Forum (2016). doi:10.1111/ cgf.12982.

[BTS∗17] BERGER M., TAGLIASACCHI A., SEVERSKY L. M., ALLIEZ P., GUENNEBAUD G., LEVINE J. A., SHARF A., SILVA C. T.: A survey of surface reconstruction from point clouds. Computer Graphics Forum 36 (2017). doi:10.1111/cgf.12802.

[Gra98] GRAY A.: Modern differential geometry of curves and surfaces with mathematica (second edition). Computers & Mathematics with Applications 36 (1998). doi:10.1016/s0898-1221(98)91133-6.

[KSNS07] KALOGERAKIS E., SIMARI P., NOWROUZEZAHRAI D., SINGH K.: Robust Statistical Estimation of Curvature on Discretized Surfaces. In Geometry Processing (2007), Belyaev A., Garland M., (Eds.), The Eurographics Association. doi:/10.2312/SGP/SGP07/013-022.

[LCBM21] LEJEMBLE T., COEURJOLLY D., BARTHE L., MELLADO N.: Stable and efficient differential estimators on oriented point clouds. Computer Graphics Forum 40 (8 2021), 205–216. doi:10.1111/ CGF.14368.

[PWY∗07] POTTMANN H., WALLNER J., YANG Y. L., LAI Y. K., HU S. M.: Principal curvatures from the integral invariant viewpoint. Computer Aided Geometric Design 24 (11 2007), 428–442. doi:10.1016/J.CAGD.2007.07.004.

[HJ87] HOFFMAN R., JAIN A. K.: Segmentation and classification of range images. IEEE Transactions on Pattern Analysis and Machine Intelligence PAMI-9, 5 (1987), 608–620. doi:10.1109/TPAMI.1987.4767955.

[CP05] CAZALS F., POUGET M.: Estimating differential quantities using polynomial fitting of osculating jets. Computer Aided Geometric Design 22 (2 2005), 121–146. doi:10.1016/J.CAGD.2004.09.004.

[Pra87] PRATT V.: Direct least-squares fitting of algebraic surface. Computer Graphics (ACM) 21 (1987). doi:10.1145/37402.37420. [CGBG13] CHEN J., GUENNEBAUD G., BARLA P., GRANIER X.: Non oriented mls gradient fields. Computer Graphics Forum 32(2013). doi:10.1111/cgf.12164.

Bibliography

[LCBM21] LEJEMBLE T., COEURJOLLY D., BARTHE L., MELLADO N.: Stable and efficient differential estimators on oriented point clouds. Computer Graphics Forum 40 (8 2021), 205–216. doi:10.1111/ CGF.14368.

[LCL∗23] LACHAUD J. O., COEURJOLLY D., LABART C., ROMON P., THIBERT B.: Lightweight curvature estimation on point clouds with randomized corrected curvature measures. Computer Graphics Forum 42 (8 2023), e14910. doi:10.1111/CGF.14910.

[CSM03] COHEN-STEINER D., MORVAN J. M.: Restricted delaunay triangulations and normal cycle. In Proceedings of the Annual Symposium on Computational Geometry (2003). doi:10.1145/777792.777839.

[MLG∗20] MELLADO N.,LEJEMBLE T., GUENNEBAUD G., BARLA P., ET AL.: Ponca: a point cloud analysis library. https://github.com/poncateam/ponca/, 2020.

[PC24] POUGET M., CAZALS F.: Estimation of local differential properties of point-sampled surfaces. In CGAL User and Reference Manual, 5.6.1 ed. CGAL Editorial Board, 2024. URL: https://doc.cgal.org/5.6.1/Manual/packages.html#PkgJetFitting3.

[DGt] DGtal: Digital geometry tools and algorithms library. http://dgtal.org.

[WEW∗22] WINIWARTER L., ESMORIS PENA A. M., WEISER H., ANDERS K., MARTINEZ SANCHEZ J., SEARLE M., HOFLE B.: Virtual laser scanning with helios++: A novel take on ray tracing-based simulation of topographic full-waveform 3d laser scanning. Remote Sensing of Environment 269 (2022). doi:https://doi.org/10.1016/j.rse.2021.112772.

[KMJ∗19] KOCH S., MATVEEV A., JIANG Z., WILLIAMS F., ARTEMOV A., BURNAEV E., ALEXA M., ZORIN D., PANOZZO D.: Abc: A big cad model dataset for geometric deep learning. In The IEEE Conference on Computer Vision and Pattern Recognition (CVPR) (June 2019).

[Yuk15] YUKSEL C.: Sample Elimination for Generating Poisson Disk Sample Sets. Computer Graphics Forum (2015). doi:10.1111/ cgf.12538.

[GG07] GUENNEBAUD G., GROSS M.: Algebraic point set sur faces. ACM Transactions on Graphics 26 (7 2007). doi:10.1145/1276377.1276406.

Bibliography

[BM16] BOULCHA.,MARLETR.: Deep learning for robust normal estimation in unstructured point clouds. In Eurographics Symposium on Geometry Processing (2016), vol.35.doi:10.1111/cgf.12983.

[QSMG17] QI C. R., SU H., MO K., GUIBAS L. J.: Pointnet: Deep learning on point sets for 3d classification and segmentation. In Proceedings- 30th IEEE Conference on Computer Vision and Pattern Recognition, CVPR 2017 (2017), vol. 2017-January. doi:10.1109/CVPR.2017.16. 10

[GKOM18] GUERRERO P., KLEIMAN Y., OVSJANIKOV M., MITRA N. J.: Pcpnet learning local shape properties from raw point clouds. Computer Graphics Forum 37 (2018), 75–85. doi:10.1111/CGF.13343.

[ZNZ∗25] ZHANG J., NIE M., ZOU C., LIU J., LIU L., CAO J.: PointNorm-Net: Self-Supervised Normal Prediction of 3D Point Clouds via Multi-Modal Distribution Estimation . IEEE Transactions on Pattern Analysis & Machine Intelligence 47, 08 (Aug. 2025), 6515–6530.doi:10.1109/TPAMI.2025.3562051.

[ZLD∗21] ZHU R., LIU Y., DONG Z., WANG Y., JIANG T., WANG W., YANGB.: AdaFit:Rethinkinglearning-based normal estimation on point clouds. In 2021 IEEE/CVF International Conference on Computer Vision (ICCV) (2021), pp. 6098–6107. doi:10.1109/ICCV48922.2021.00606. 10, 19

[LZW∗22] LI K., ZHAO M., WU H., YAN D.-M., SHEN Z., WANG F.-Y., XIONG G.: GraphFit: Learning multi-scale graph-convolutional representation for point cloud normal estimation. In Computer Vision ECCV 2022:17th European Conference,Tel Aviv, Israel, October 23–27, 2022, Proceedings, Part XXXII (Berlin, Heidelberg, 2022), Springer Verlag, p. 651–667. doi:10.1007/978-3-031-19824-3_38.10, 19

[ZJW∗23] ZHOU J., JIN W., WANG M., LIU X., LI Z., LIU Z.: Improvement of normal estimation for point clouds via simplifying surface fitting. CAD Computer Aided Design 161 (2023). doi:10.1016/j.cad.2023.103533. 10, 19

Bibliography

[LOM20] LENSSEN J. E., OSENDORFER C., MASCI J.: Deep iterative surface normal estimation. In Proceedings of the IEEE Computer Society Conference on Computer Vision and Pattern Recognition (2020). doi:10.1109/CVPR42600.2020.01126.

[BSG20] BEN-SHABAT Y., GOULD S.: Deepfit: 3d surface fitting via neural network weighted least squares, 2020. doi:10.1007/978-3-030-58452-8_2. 10, 19

[CZB∗22] CAO J., ZHU H., BAI Y., ZHOU J., PAN J., SU Z.: Latent tangent space representation for normal estimation. IEEE Transactions on Industrial Electronics 69 (2022). doi:10.1109/TIE.2021.3053904. 10

[LLC∗22] LI Q., LIU Y.-S., CHENG J.-S., WANG C., FANG Y., HANZ.: HSurf-Net: Normal estimation for 3D point clouds by learning hyper surfaces. In Advances in Neural Information Processing Systems (NeurIPS) (2022), vol. 35, Curran Associates, Inc., pp. 4218–4230.

[LFS∗23] LI Q., FENG H., SHI K., GAO Y., FANG Y., LIU Y.-S., HANZ.: SHS-Net: Learning signed hyper surfaces for oriented normal estimation of point clouds. In Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition (CVPR) (Los Alamitos, CA, USA, June 2023), IEEE Computer Society, pp. 13591–13600.doi:10.1109/CVPR52729.2023.01306.10

[LSY24] LYU B., SHEN L.-Y., YUAN C.-M.: Igf-fit: Implicit gradient field fitting for point cloud normal estimation. Graphical Models 133 (2024), 101214. doi:10.1016/j.gmod.2024.101214.

Bibliography

[RB25] RAUCH L., BRAML T.: Rohbau3D: A shell construction site 3d point cloud dataset. Scientific Data 2025 12:1 12 (8 2025), 1478–. doi:10.1038/s41597-025-05827-7

[HJP∗24] HANSEN L. H., JENSEN S. B., PHILIPSEN M. P., MØGELMOSE A., BODUM L., MOESLUND T. B.: OpenTrench3D: A photogrammetric 3d point cloud dataset for semantic segmentation of underground utilities. In 2024 IEEE/CVF Conference on Computer Vision and Pattern Recognition Workshops (CVPRW) (2024), pp. 7646–7655. doi:10.1109/CVPRW63382.2024.00760. 17

[HSL∗17] HACKELT., SAVINOVN., LADICKYL.,WEGNER J.D., SCHINDLERK., POLLEFEYSM.: SEMANTIC3D.NET: A NEW LARGE-SCALE POINT CLOUD CLASSIFICATION BENCHMARK. ISPRS Annals of the Photogrammetry, Remote Sensing and Spatial Information Sciences IV-1/W1 (2017), 91–98. doi:10.5194/ isprs-annals-IV-1-W1-91-2017.5